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ELEMENTS

OF

PRACTICAL GEOMETRY,

FOR

SCHOOLS AND WORKMEN.

Horace Grant

BY THE AUTHOR OF

"ARITHMETIC FOR YOUNG CHILDREN," "EXERCISES FOR THE SENSES,"
"DRAWING FOR YOUNG CHILDREN," &c

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NOTICE.

THIS Treatise is intended for young workmen, and for the older pupils in schools where theoretical Geometry is not taught. It explains only the most simple and useful facts and operations which every person ought to know. A great part of it does not require a knowledge even of Arithmetic; and no kind of explanation has been used that is likely to be found difficult by a beginner.

In learning the subject, each portion should be read carefully several times; and the drawings should also be copied more than once, of as large a size as possible.

When the treatise is used by a teacher, he should explain each part, and make large drawings before the pupils. He should then question the pupils upon every part, and should cause them to make drawings on a black board or slate, or upon paper, as large as is convenient.

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INTRODUCTION.

I.—WHAT GEOMETRY MEANS; AND ITS PARTS OR DIVISIONS.

1. **Geometry** is the name for what we know about the shapes and forms* of things.

2. We learn the *names* of shapes and forms, such as balls, cubes, squares, circles, triangles, and many others,—no matter what they are made of, or how large or small they are. This naming is part of Geometry.

3. We learn what *parts* these shapes and forms have; and how to make up or to draw such shapes and forms.

4. We learn how to *measure* and *calculate* objects of many different shapes and forms. Thus, we find out how much wood or stone a log of wood or block of stone contains; how many acres there are in a field; how much carpet is needed for a room; what is the distance across a river which we cannot get over; and the height of a tree or steeple which we cannot get up to measure: also how to make maps of countries, rivers, lakes, and seas; and many other useful things. All these are parts of Geometry, or are taught by Geometry.

5. We learn to *compare* together different shapes and forms, so as to find if they are of the same size or not:—as

---

* Solid Bodies have shape; Figures, such as triangles, squares, and circles, have form.

whether a thick short log of wood contains as much timber as a long thin one; if a certain plank contains enough wood for a box or table we wish to make; how much plastering, or painting, or paper-hanging is required for a room; what square, or circle, or oblong is equal to a certain triangle; what size of ball is equal to a certain cube or square solid; and many other things very useful in most trades.

6. There are also many other parts of Geometry of much use to carpenters, smiths, bricklayers, masons, surveyors, engineers, seamen, astronomers, and to almost all other trades and professions, some of which are explained in this treatise.

7. Geometry has two principal parts or divisions, *Solid Geometry*, and *Plane Geometry*.

8. SOLID GEOMETRY. The naming, measuring, and comparing the shapes of solids, such as bricks, boxes, books, balls, pillars, stems of trees, extinguishers, and many other shapes, is called Solid Geometry or the Geometry of Solids.

9. We also learn by Solid Geometry how much a box, or cistern, or ship, or any hollow body will hold. This is called the capacity of the box or other hollow body.

10. All solids have outsides or surfaces. Some outsides are flat, such as those of bricks, books, and floors. These flat outsides are called plane surfaces or planes. Other solids have rounded surfaces or outsides, as balls, trunks of trees, basins, and many others. These are called curved surfaces. Many objects have rough surfaces, but only the flat or plane surfaces, and the rounded surfaces, are treated of in Geometry, because rough or irregular surfaces cannot be named or measured exactly.

11. PLANE GEOMETRY. The forms of the most common and useful plane surfaces are triangles, squares, oblongs,

circles, and ovals. The part of Geometry which explains these is called Plane Geometry.

12. Lines of all kinds, including the boundaries of surfaces and edges of solids, are a subject of Geometry; and the part which relates to lines is sometimes called Lineal Geometry, but it is included in Plane Geometry.

13. Geometry, then, is one word or name for a great many different things, all of which have something to do with forms and shapes. These things are so much connected together, that many of them cannot be understood unless others of them are first explained. A complete collection of these things or of this kind of knowledge, when it is put into good order and thoroughly explained, is called the Science of Geometry, or Theoretical Geometry.

14. Practical Geometry. Those parts of Geometry which are most useful for the common purposes of life, are picked out from the rest and explained, and their uses to workmen and others are shown. These are called Practical Geometry.

II.—WORDS AND NOTIONS MOST COMMON AND IMPORTANT IN GEOMETRY.*

15. There are some words and notions so important that nothing practically useful in Geometry can be known unless they are first understood.

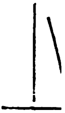
16. Lines. A line has length but no breadth, or the least possible breadth that will allow it to be seen; we can think of it as if it had no breadth.

---

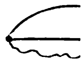
* The learner when going through this treatise, should draw the figures as large and correctly as possible.

17. Edges and thin marks or scratches of all kin called lines.

18. Kinds of Lines. There are three sorts of straight, curved, and crooked, or irregular.



19. 1st. Straight Lines are often called Right Lines.



20. A straight line is the shortest way between two points.

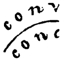
21. 2nd. Curved Lines are rounded: they a straight in any part.

a. Some are curved in one manner only.

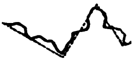
b. Others in several ways or degrees

c. Serpentine lines are like a serpent

d. Spiral lines go out from a middle point and it; and the more they are lengthened the farther they go from the middle point.



22. The outer part or side of a curved line is called convex; the inner side is called concave

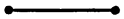


23. 3rd. Crooked or Irregular Lines cannot b sured exactly, but they can be measured near enough for common purposes, by means of several straight lines, as in this diagram, where the four dotted straight lines ar nearly the length of the irregular line. Such lines occur in measuring land, and in making maps, and Ge shows the best way to measure them and draw them.

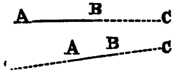
24. A Point is the smallest portion of space that seen or thought of. It has a place but no size. Th

of a line are points; also the place where two lines cross.

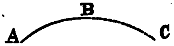
25. Two points are said to be *joined* when a straight line is drawn from one to the other.



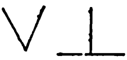
26. When a line is lengthened in the same direction, it is said to be *produced* or continued. The line between A and B is produced or continued to C.



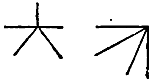
27. Curved lines may be produced.



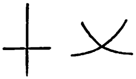
28. Lines are said to *meet* when one line ends at some point of another line, but does not cross it



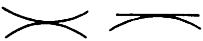
Several lines may meet, thus



29. Lines are said to *intersect*, or to *cut*, when they cross each other.

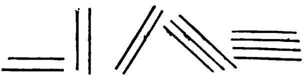


30. One line is said to *touch* another when it goes to the other and away again without intersecting it. One or both of these lines must be curved. Two lines can touch in one point only.

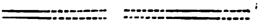


31. Positions of Lines. Straight lines may lie in several different ways, and are named differently on that account, viz.,—parallel, vertical, perpendicular, horizontal, and inclined.

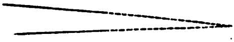
32. Parallel Lines. When lines lie side by side, and would not meet though produced to any distance, they are called parallel lines, or for shortness,



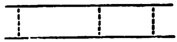
parallels. Any one is said to be parallel to the other or others.



33. These lines are not parallel, because they meet at one end when they are produced.

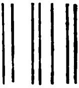


34. The opposite points of parallels are always equally distant.



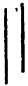
35. Railings, lines of the panels of a door, edges of a book or of paper, lines of print, railway bars, &c., are parallel lines.

36. Any number of lines may be parallel (as all the lines of a page of print), and they may lie at any distance from each other.



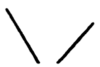
37. Vertical Lines or Plumb Lines stand upright from the ground like a post or pillar, or the corner of a house, or they hang down like a string with a weight at its end, such as is used by bricklayers, masons and carpenters, to show if their work is correct.

38. Perpendicular Lines lie direct between top and bottom of a paper or object, as these. A perpendicular line is sometimes a vertical line, sometimes it is not so. The side edge of a book or slate is always perpendicular to the bottom edge, although we hold the book or slate in different ways. But when we hold the side edge upright from the table or ground, it is vertical as well as perpendicular.

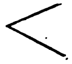


39. Horizontal Lines lie across, so They are sometimes called flat or level lines. Lines on paper that are parallel to the top and bottom, are horizontal. Also the top and bottom edges of a door or window, or picture frame, or book, or slate. A straight rod or a stretched string on level ground, or on the surface of water, is also horizontal.

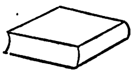
40. Slanting Lines are such as are not vertical, or perpendicular or horizontal. They are also called *inclined* lines, or *oblique* lines.



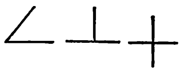
41. Angles. When two lines meet at one point, thus, the opening between them is called an angle. The lines are said to contain the angle, and to be sides or legs of the angle.



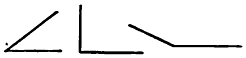
42. Edges that meet, such as those of boards, books and walls, make angles.



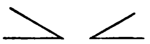
43. Two lines can form one, two or four angles.



44. Names of Angles. Angles are named according to their bluntness or sharpness; that is, according as their legs, when of equal length, are more or less wide apart. The hands of a clock form angles whenever one hand does not cover the other; and the angles are different when the ends of the hands are more or less distant.



45. One angle is said to be *equal* to another when the legs of both are equally open. Such angles are equally sharp. These angles are equal



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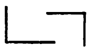

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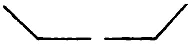
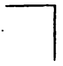
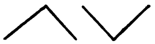
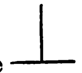


46. Right Angle. A right angle is formed by a perpendicular meeting a horizontal line. Any angles equal to these are right angles. Any one of these last pairs of lines is said to be perpendicular to the line that meets it.

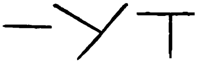




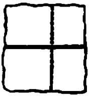
47. When one line meets another so as to form equal angles, thus, these two angles are



right angles, and either of the lines is said to be perpendicular to the other.

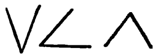


48. Fold a piece of paper so as to make a straight crease across it (see the dark line), then fold one half of this crease over on the other half, so as to make a second crease (as shown on the dotted line). Four right angles will then be made, which will cover each other exactly, if cut out and laid on each other, or if folded over each other.

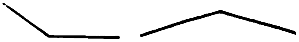


49. Most of the angles of doors, windows, rooms, and parts of buildings and furniture are right angles.

50. Acute angle. An acute angle is sharper or smaller than a right angle.

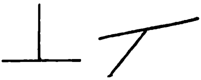


51. Obtuse Angle. An obtuse angle is blunter or larger than a right angle.

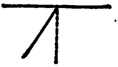


52. Every angle must be either right, or acute, or obtuse.

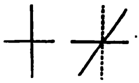
53. Adjacent Angles adjoin each other, or are next to each other.



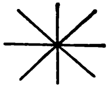
Two adjacent angles must be right angles, or equal to two right angles.



54. When one line intersects another, the angles must be four right angles, or two acute and two obtuse angles equal together to four right angles.

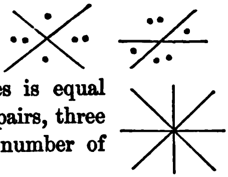


55. Any number of lines intersecting through one point, make angles equal to four right angles.

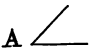


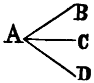
56. Vertical Angles. When one line crosses or intersects another, the pairs of angles whose points are opposite

(viz., . and . , and . . and . .), are called vertically opposite, or for shortness, vertical angles. Each angle of a pair of vertical angles is equal to the other. There may be two pairs, three pairs, four pairs, or any greater number of pairs of vertical angles.



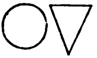
57. Angles are usually marked or referred to by letters. This angle is called the angle A. The upper one of these angles is called the angle BAC or CAB; the lower one CAD or DAC, the middle letter being that at the angular point.

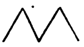


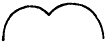


58. FIGURES. A figure is a surface, or a part of a surface, quite enclosed by lines or edges, which are called its boundaries.*

59. This page is a figure, so are these. So also are the sides of boxes, and rooms, and slates, and books, the tops of tables, &c. The following drawings are not figures, because the lines do not completely enclose the spaces.

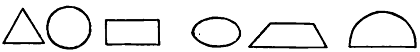






60. A figure has length and breadth; it cannot have any thickness; if it had it would become a solid, and be more than a figure.

61. PLANE FIGURES. This page and all these figures are called plane figures because



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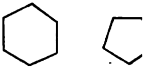
* The word Figure, in Arithmetic, signifies a mark for a number, as 2 or 5, and has quite a different meaning from the same word in Geometry. Drawings made to explain anything, such as any of those in this book, are also called Figures, but Diagrams is their correct name.

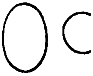
they are plane-bounded surfaces. So also are floo ceilings, sides of rooms, panels of doors, &c.

62. Figures can also be made on *curved* surfaces, as wh part of the surface of a ball, or bowl, or cup, is enclos by a line or lines. Figures can also be made on rough irregular surfaces, as on rough stones; they are call irregular figures, and are not particularly examined Geometry. None but plane figures are treated of Elementary Geometry. Those on curved surfaces are not useful, and are more difficult to measure perfectly.

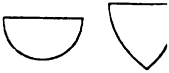
63. **Rectilineal Figures.** Figures made with straight (or right) lines only, as triangles, squares, &c., are called right-lined or rectilineal figures.



64. **Curvilineal Figures** are those which are made with curved lines only, as circles and ovals.



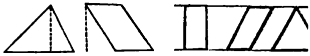
65. Figures can be made partly with right lines and partly with curved lines. These are sometimes called *mixtilineal* or mixed-line figures.



66. Figures can also be made with irregular lines.

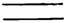
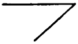
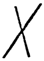
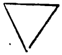
67. A surface that is enlarged in the same direction said to be *produced*, like a piece of flat doubled paper wh unfolded.

68. The *altitude* of a figure is its perpendicular height, or the shortest distance between two parallels, which the fig would lie between exactly.

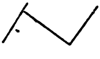
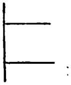


69. **Plane Rectilineal Figures.** A figure cannot

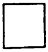
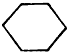
made by one straight line, or by two straight lines. Three straight lines are the least number that will enclose a space. And the figure so formed must have three angles.

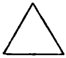
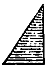
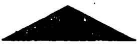
No one of these threes of lines is a figure.

70. Figures can also be made with any number of straight lines greater than three. Such figures have different names, according to the number of their boundary lines.

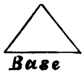



71. Triangle. A figure made by three straight lines is called a triangle, because it has three angles. Tri means three; triangle, therefore, means a three-angled figure.

72. The lines are called *sides* of the triangle.

73. The side that is lowest is generally called the *base line* or the *base;* but any side may be considered as the base.



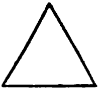
74. The angle opposite the base is called the *apex* or *vertex* of the triangle, or the vertical angle. The words apex and vertex mean top.

75. The other two angles are called the angles at the base.

76. Kinds of Triangles. Though a triangle must have three sides and three angles, the sides may be of different lengths, and the angles of different sizes. Several different names are given to triangles to show the comparative length of their sides, and size of their angles.

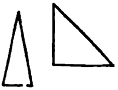
77. 1st. Triangles are distinguished and named according to the comparative length of their sides.

78. The Equilateral Triangle has three equal sides, equi means equal; lateral means sided. It has also three equal angles.

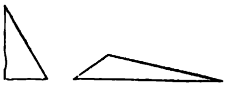


79. All equilateral triangles are of precisely the same form, and have the same angles, although some cover more space than others. Figures which have the same form are called *similar* figures.

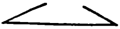
80. The Isosceles Triangle has two equal sides only. The remaining side of an isosceles triangle is called the base.



81. The Scalene Triangle has three unequal sides: but any two of its sides must be longer than the third.



82. A triangle cannot be made with three lines unless any two of them are longer than the third, as may be shewn by a folding rule.

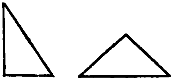


83. 2nd. Triangles are also distinguished and named according to the size of their angles.

84. An Acute Angled Triangle has three acute angles.

85. It may be equilateral, or isosceles, or scalene. Its angles may be all equal. It is then called an equiangular triangle, which is also equilateral.

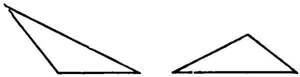
86. A Right Angled Triangle has one right angle and two acute angles. It cannot have more than one right angle: and it cannot have an obtuse angle.



It may be isosceles or scalene.

The longest side, which is always opposite the right angle, is called the hypothenuse.

87. An **Obtuse Angled** Triangle has one obtuse and two acute angles. It cannot have a right angle; or more than one obtuse angle.



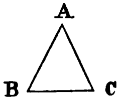
It may be isosceles or scalene.

It may have two angles equal; or all unequal.

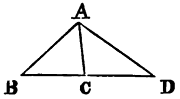
88. The largest angle of a triangle is opposite to its longest side. A longer side is opposite to a larger angle than a shorter side. Equal sides have equal angles opposite them.

89. The three angles of any triangle are equal, together, to two right angles. (See No. 328).

90. Triangles are usually marked by three letters thus



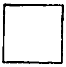
The left of these two adjacent triangles is called the triangle ABC, or BAC, or CAB. The right one is called ACD, or ADC, or DAC.



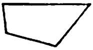
91. **Four-Sided Figures.** Four-sided figures are also called *quadrilateral* figures, as quadrilateral means four-sided. They are also called *quadrangular* figures or quadrangles, which means four-angled figures, because a four-sided figure must have four angles.

92. They are named according to the comparative length of their sides, and the size of their angles.

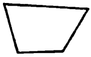
93. They may have all their sides equal, as



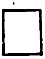
or all unequal



and all the angles equal or all unequal; or two or three only of their sides and angles may be equal.

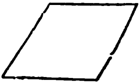


94. The Square has four equal sides and four right angles, therefore it is said to be equilateral and equiangular. Its opposite sides are parallel.



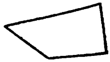
95. The Oblong or Rectangle has only each opposite pair of sides equal. All its angles are right angles.

96. The Rhomb (Lozenge or Diamond) has equal sides, and the opposite pairs parallel. But only each pair of opposite angles are equal, one pair being acute, the other obtuse.

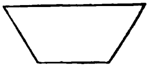


97. The Rhomboid has only the opposite pairs of sides equal and parallel, and only the opposite pairs of angles equal; one pair being acute, and the other obtuse.

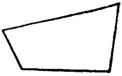
98. Trapezium. All other four-sided figures are called trapeziums. They may have two sides equal



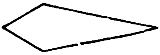
or three sides equal



or all unequal

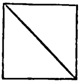


or pairs of sides may be equal but not opposite pairs. One pair of sides may be parallel, not more, or none may be parallel. They may have two or three angles equal, or none.

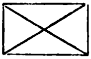


99. Rectangle. Any four-sided figure that has four right angles, is a rectangle, but the name is most frequently given to the oblong.

100. A **Parallelogram** is any four-sided figure that has its opposite sides parallel. Parallelogram means parallel-drawn; squares, oblongs, rhombs, and rhomboids, are parallelograms.



101. **The Diagonal** is a line that crosses obliquely from angle to angle

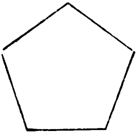


A four-sided figure can have two diagonals. A diagonal divides a parallelogram into two equal triangles.

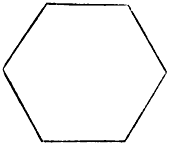
102. **Polygons.** Any rectilineal (right-lined) figure that has more than four sides is called a polygon. Poly means many, and gon means angle. Polygons may have any number of sides and angles; but the number of the sides and angles must be the same in any one polygon.

103. A polygon may have all its sides equal, and all its angles equal; in which case it is called a regular polygon. Or a part only of its sides or angles may be equal; it is then said to be irregular.

Pentagon. A five-sided figure is called a pentagon.

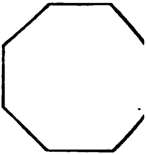


Hexagon. A six-sided figure is called a hexagon.



Heptagon. A seven-sided figure is called a heptagon.

Octagon. An eight-sided figure is called an octagon.



Nonagon. A nine-sided figure is called a nonagon.

Decagon. A ten-sided figure is called a decagon.

Dodecagon. A twelve-sided figure is called a dodecag

104. The first syllable of each of these names is t Greek word for a number, namely, for 6, 7, 8, 9, 10 and The last syllable gon, is the Greek for a knee or angle.

105. When we speak of a pentagon, or any other po gon, we usually mean a regular one, unless we say to t contrary.

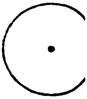
106. Triangles are by far the most useful of the rec lineal figures. All other figures can be divided into triangl and measured by means of triangles.

107. Four-sided figures are next in importance triangles; and of much greater use than polygons.

108. Curvilinear Figures, or figures made wi curved lines on a plane. A great variety of figures c be made having one curved line as a boundary, but o two, namely the circle, and the oval (or ellipse), are much practical use.

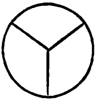
109. Circle. The circle has one curvilineal or curved-line boundary, which is called its *circumference.*

110. The plane enclosed by the circumference is called the *disc.*

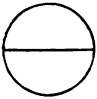


111. The Centre of the circle is a point equally dist from every part of the circumference.

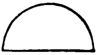
112. Any line from the centre to the circumference is called a *radius* or ray. (Radius is a Latin word, and its plural is radii, not radiuses; we say two radii, three radii). All the radii of a circle are of the same length.



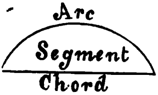
113. Any straight line which crosses a circle, and passes through its centre, is called a *diameter*. A diameter is made of two radii: it is the longest line that can be made in a circle.

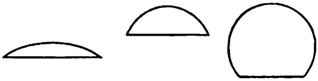


114. A circle is divided by a diameter, into two halves, called *semicircles*.

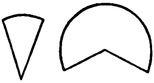


115. A part cut off a circle by one straight line, is called a *segment* of a circle. Its curved line is called an *arc* or arch. Its straight line is called a *chora* or cord. A chord is opposite to, or *subtends* an arc. A segment may be a semicircle, or any larger or smaller part of a circle. It is bounded by a right line and a curve.

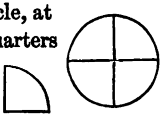




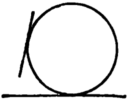
116. A Sector is a part cut out of a circle by two radii. It is bounded by two straight lines, and one curved line.



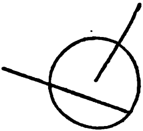
117. When two diameters cross a circle, at right angles, they divide it into four quarters or sectors, which are called *quadrants*.



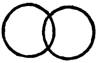
118. Any line touching a circle, but not going into it, is called a *tangent*.

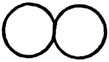


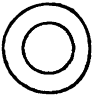
119. Any line cutting through a circle, is called a *secant*.

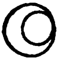


120. Two circles are said to be *concentric* when they have the same centre; and *eccentric* when they have different centres.

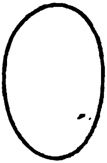




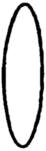




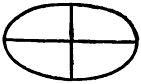
121. **Oval** and **Ellipse**. The ellipse and oval have a circumference and a disc, but no central point from which every radius is equal. All parts of the circumference of these figures are not curved alike.

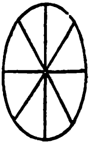


122. Their breadth compared with their length, may vary very greatly.



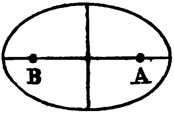
123. They have one diameter longer than any other that can be made in them; and one shorter than any other. Each of these diameters is called an *axis*. The longer axis is called the *transverse* axis—the shorter, the *conjugate* axis. All other diameters must be shorter than the transverse, and longer than the conjugate axis.



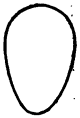


124. The ellipse is sometimes called an oval; but there is a difference between an ellipse and

an oval. The oval is formed of parts of different circles joined together. The ellipse is not made up of parts of circles. It is curved differently from a circle in every part; and is a more perfect figure. But the difference between an oval and an ellipse, if they have the same axes, is very slight.



125. There are two points within an ellipse, by means of which it can be drawn, as is shown in No. 310. Each of these points (A and B) is called a *focus;* in the plural, foci.



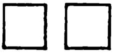
126. The Egg-Shaped Oval and Ellipse have one end rounder than the other. (See 309, 312—14.)

127. Various curve-lined figures can be made on a ball or any other curved surface, as a watch glass. The surface of such figures is not a plane.

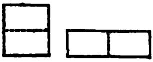
128. Irregular Figures. Figures bounded by irregular lines are not included directly in Geometry, as they cannot be distinguished, or named, or measured, with perfect accuracy. When the regular figures above-noticed are understood, the irregular ones can be estimated or measured with sufficient accuracy for common purposes, as is shown in No. 332.

129. The Area of a figure is the quantity of space which it covers.

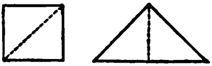
130. Two figures are *equal* if they have the same area, as



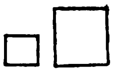
also,



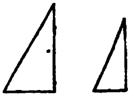
and,



131. Figures are *similar* if they have the same form, as these

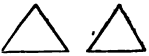


or these,

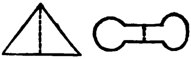


They may be equal or unequal in area.

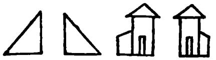
132. Figures are *identical* when their area and form are the same; that is, when they are equal and similar.



133. A figure is *symmetrical* if one half will fold over on the other half, and cover it exactly.



134. Two figures are *symmetrical* when, if one be folded over, or turned over, upon the other, they will cover each other exactly, as these pairs



135. Planes may be parallel to each other, as the opposite walls of a room; or horizontal, as the surface of a pond; or vertical, as the walls of a house. An edge of several planes may meet at any angle, as a door more or less open with the adjacent wall. Two or more planes may also be conceived to intersect.

136. Solids. A solid must have length, breadth, and thickness.

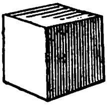
137. Solid objects have an infinite variety of shapes; but Geometry teaches how to measure and calculate a few shapes only, that are very simple or regular, and are easily distinguished from all others. When we know how to measure and compare objects of these regular shapes, all other shapes, however irregular, can be calculated or estimated

by means of the former, not quite perfectly, but with sufficient accuracy for all useful purposes.

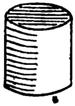
138. The chief shapes that Geometry treats of are cubes, prisms, pyramids, cylinders, and cones.

139. Solids are distinguished by the kind or number of parts of their outside or surface. These outside parts are said to bound or enclose the solids.

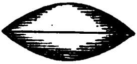
140. The boundary or outside of some solids is separated by edges or lines, into distinct portions. These portions are either wholly planes, as in a square box or cube, which are called *faces* or sides:



or are partly planes and partly curved surfaces, as those of a cylinder or cork.



Or they consist of several curved surfaces united, as in a plum stone or cockle shell.



141. Some solids, as balls and eggs, have only one continuous surface, without lines, edges or angles.

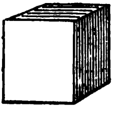
142. Solids have

I. Plane faces or surfaces.

II. Curved surfaces.

III. Surfaces, partly plane, partly curved.

1.—Solids with Plane Faces.



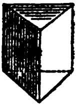
143. The Cube is a solid with six square faces of the same size. It has eight solid angles, each formed by the meeting of three right angles belonging to three adjacent sides. The lowest face is called the *base;* the upper

face, the top. Dice, and perfectly square boxes are of this shape.

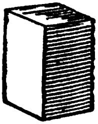
144. The cube is the standard of measurement for measuring solids; that is, we calculate or state what cube or cubes of a known size, the solid which we measure is equal to: because the cube is the solid which is most easily understood and calculated. Thus, when we desire to name the bulk or solid contents of a ball (or any object), we make the proper calculation, and find (if the solid be small), that it is equal to so many cubic inches, that is, cubes which have a square inch for each face: or if the solid be large, to cubes that have a square foot, or a square yard for each face.

145. The PRISM is a solid with five faces, or any greater number. These faces must be rectilineal figures. A prism has a face at the bottom and one at the top of the same shape and area, which are parallel, and are called bases. The remaining faces are of the same number as the edges of a base, and they must be parallelograms.

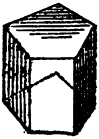
The THREE-SIDED OR TRIANGULAR PRISM has triangles for bases.



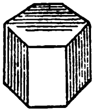
The FOUR-SIDED OR QUADRANGULAR PRISM has squares for bases. It may also have squares for sides, in which case it is a cube.



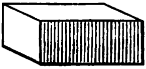
The FIVE-SIDED OR PENTAGONAL PRISM has pentagons for bases.



The SIX-SIDED or HEXAGONAL PRISM has hexagons for bases.

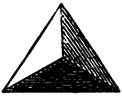


146. The PARALLELOPIPED is a quadrangular prism, which usually has oblongs for sides and bases, and each opposite pair of faces must be parallel. Bricks are of this shape; and so, usually, are books, blocks of stone, logs of wood, rooms, houses, and boxes. Manufactured articles are most frequently made of this shape. Any solid that has three pairs of parallel sides, may be called a parallelopiped.

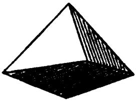


147. PYRAMIDS have rectilinear figures for bases; but their sides, which are triangles, diminish upwards till they unite in one point or solid angle, called the apex.

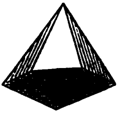
THREE-SIDED or TRIANGULAR PYRAMIDS have triangular bases and sides.



FOUR-SIDED or QUADRANGULAR PYRAMIDS have four sides and a square base.



FIVE-SIDED or PENTAGONAL PYRAMIDS have five sides and a pentagon for base.



SIX-SIDED or HEXAGONAL PYRAMIDS have six sides and a hexagon for base.

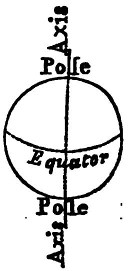
Pyramids may have any larger number of faces, a base must have as many sides.

II.—Solids with Curved Surfaces.

148. Sphere. The sphere, globe, or ball has a sur rounded equally in every part, and not separated i distinctly marked parts, like the cube and other pla faced solids.

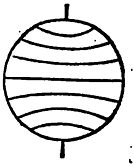
149. It has a middle point or *centre* equally distant fro every part of the surface, which no other solid has. A lin from this central point to the surface is called a *radius.*

150. Any line passed through the centre of the sphere, from one point of the surface to the opposite point, is called an *axis* of the sphere, and the sphere would spin evenly round it.



151. The two extremities of the axis are called *poles.*

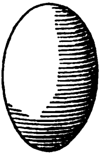
152. A line drawn round the sphere, equally distant from the poles, is called the *equator,* which means an equal divider. It is also called a *large circle* of the sphere.



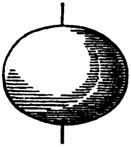
Every circle round the sphere, which is not an equator, is less than that circle, and is called a *small circle* of the sphere.

153. A broad ring or belt round a sphere is called a *zone*.

154. SPHEROID. A spheroid is a rounded solid, something like a sphere; but it is either lengthened out at two poles, or flattened at two poles.

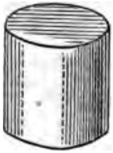


155. When it is lengthened out, it is called a *prolate spheroid* or an *ellipsoid*, because its outline is like an ellipse, as plums, cocoa nuts, &c.,

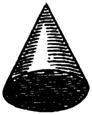


156. When it is flattened at the poles, like an orange or apple, it is called an *oblate spheroid*. The world is an oblate spheroid.

III.—SOLIDS WITH SURFACES PART PLANE AND PART CURVED.



157. CYLINDER. The cylinder has two parallel bases, which are circles, and a curved surface between them, which joins them. Corks, pillars, rollers, and stems of trees are of this shape.

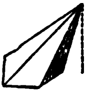


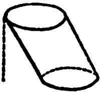
158. CONE. The cone has a circular base, and a curved surface which contracts as it rises, till it ends in a point, like an extinguisher.

159. An irregular cylinder and cone can be made on oval bases, with a corresponding change of the curved surface.

160. Right and Oblique Solids. A parallelopiped, prism, pyramid, cylinder, or cone, that stands upright o its base, is called a right parallellopiped, a right prism, &c

161. When either of these solids will not stand upright, or vertically, on its base, but leans to one side, it is called an oblique prism, or cylinder, &c.





162. The Altitude of a solid is its vertical height, o the length of a vertical line from its base to its highest sid or point. See dotted lines in last two diagrams.

163. Regular Solids. A solid that has all its faces o the same form, and inclined at the same angles to eac other, is called a *regular solid.* There are only five of thes regular solids, namely :—

1. The Tetragon, or three-sided pyramid with four equ equilateral triangles for faces.
2. The Hexahedron, or cube, with six equal square face
3. The Octohedron, or double four-sided pyramid, wit eight equal equilateral triangles for faces.
4. The Dodecahedron, with twelve equal pentagons fo faces.
5. The Icosahedron, with twenty equilateral triangl for faces.

ELEMENTARY PRACTICAL GEOMETRY.

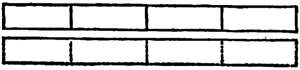
164. The two most important operations in Practica Geometry, are to make straight lines and circles.

165. To make Straight Lines. If we have a *rule,* o any *straight edge,* straight lines can be ruled against it, o

paper, with a pencil, pen, or chalk point; or scratched on wood or metal, with a hard point.

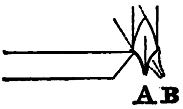
166. The point which we rule with should be held close to the straight edge; and the hand should be kept in the same position, and moved steadily in the same direction.

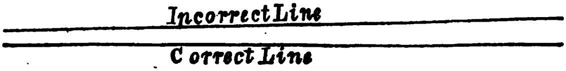
167. To find if a Ruling Edge is straight. Rule a line with it: then turn the rule over on its edge and rule a second line parallel to the first, and very near it. If the edge is correct, the two lines will agree, and be parallel in every part: if the edge is incorrect, the two lines will not agree.



168. A String or Rope stretched tightly between its ends, lies in a straight line. Gardeners, bricklayers and other workmen, work in straight lines by this guide. Painters chalk a string and mark a faint chalk line with it, by stretching it over work which they could not rule.

169. When ink lines are ruled with a pen against a straight edge that touches the paper, the ink is apt to run and blot the lines. On this account the bottom edge of the flat ruler is bevelled, or planed off, so that the pen's point shall not touch the lower edge. A round (cylindrical) ruler is also used with a pen. But great care must be taken that the pen be always held in the same position; for if the hand get inclined, even slightly, in ruling the line, the point of the pen will gradually move as from A to B in the above diagram, and the line will be somewhat curved.





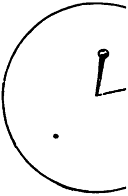
170. When it is of importance that the lines perfectly correct, a steel ruling pen is used, which will make the finest lines against a straight edg blotting.

171. If a drawing is intended to be coloured, should be used in place of writing ink, because runs when the paper is washed over with colour.

172. To make Circles. Circles can be made ways.

173. 1st. By Tracing. Lay a penny on the mark or trace a circle round it with a pencil or sh Larger circles can be made in this way with the cup or basin; and smaller circles with a half shilling.

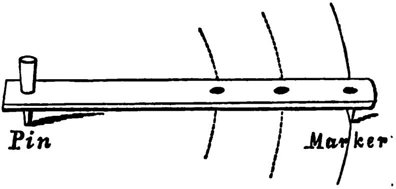
174. 2nd. With Pin and Thread. Stick a pin or round nail where the centre is to be. Tie one end of a thread round it. Tie the other end of the thread round the pencil, close to the point, at the proper distance from the centre; and carry the pencil round, so as to make a curved line, which will b The pencil must be held perfectly upright and ste



175. Circles of a large size are made in this wa ground, by tying a string or cord to a stick or sta

176. 3rd. With Rod and Pin. Circles of a larg also be made with a rod or lath of wood, by fixin

to the ground with a pin, so that the wood shall travel round the pin, like the hand of a clock. A marker of some kind must be fixed on the opposite end of the rod, to scratch or mark the circle. Holes for a marker can be made in the rod at several distances from the centre, by which smaller circles can be drawn.



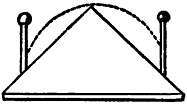
177. Circles can be drawn in the same manner on paper, by using a long narrow slip of pasteboard, which is fixed to the centre by a common pin, and has a hole for a pencil's point to go through and mark the circle.

178. 4th. WITH COMPASSES. Circles that are not very large, can be drawn most easily and correctly with a pair of compasses. In using compasses for this purpose, keep them nearly upright; roll their head slowly round in the hand, and press on them equally but not heavily.

179. It is often convenient to have compasses with a pencil leg, as common compasses will only scratch a line. If there be no pencil leg, a bit of lead pencil tied firmly on one leg, will answer the purpose.

180. Circles cannot be drawn well when the compasses are very wide open, or when they are nearly closed.

181. 5th. WITH AN ANGLE. A circle, or part of one, can be made if we stick two pins or nails into the paper or ground, the same distance apart as the diameter of the circle we want to draw, and then place a paper or wooden right angle, of proper size,

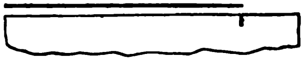


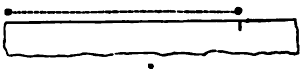
between the pins, and move it round from one pin to the other, always keeping its sides touching the pins. By making dots at the point of the angle, as it goes round, half a circle will be made. We must then put the angle through on the other side of the pins, and draw the other half of the circle in the same manner.

182. Builders frequently use this mode of drawing large arches, or parts of circles, in narrow places where they have not room to draw a circle from a centre, and they make the triangle of three rods of wood nailed together.

183. An arc of a circle less than a semicircle, can be made in this way by using an obtuse-angled triangle: an arc greater than a semicircle, by using an angle less than a right angle.

184. A LINE MADE OF THE SAME LENGTH AS ANOTHER LINE. 1st. Supposing that you want to make a line somewhere else exactly as long as this black line; lay a slip of paper which has a straight edge along this line, and mark the length of the line on the slip: then lay the slip on the place where you want the line to be drawn; make a dot at each mark of the slip, and rule a line between these dots.





185. 2nd. The line can be *measured* on a *rule* to find its length in inches, feet or yards, and the required line can be drawn of the same length.

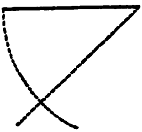
186. 3rd. If the line be long, and no measure be at hand, the length of it can be marked on a long rod, or on a piece of string or rope, and marked out by that means at the place where you want to make it.

187. Surveyors and others who have to measure great

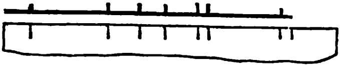
lengths, sometimes use long rods, and sometimes long chains.

188. 4th. WITH COMPASSES. Open the compasses so that their points shall be on the ends of the line; you can then mark out the length of the line wherever you desire to draw another.

189. If the new line is to be drawn from the same point or centre as one end of the given line, but in another direction (as is shown by the dotted line), place the compasses on the given line, as before, and draw an arc through the dotted line. This arc will cut off the same length as the given line.



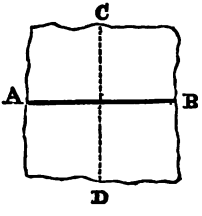
190. 5th. If the given line be divided into several parts, the length of the whole line, and of each of the parts, can be marked on a slip of paper, and taken at once to any other line.



191. The same thing could be done with compasses, but not at once.

192. TO MAKE A PERPENDICULAR LINE AND A RIGHT ANGLE.

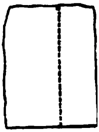
1st. BY FOLDING PAPER. Make a straight fold mark, AB, across a piece of paper; fold over one half of this line upon the other half, so as to make another fold, CD. Either of the folded lines thus made is perpendicular to the other, and each of the four angles is a right angle.



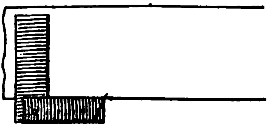
193. If the paper be carefully folded, and the angles be

cut out, right angles and perpendiculars will be procured, to compare with other angles and lines.

194. If the paper has a straight edge, fold one half of the edge on the other; thus a perpendicular and two right angles will be made.

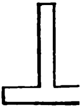


195. 2nd. WITH CARPENTER'S SQUARE. The carpenter square is a right angle made of wood and iron. One side of it is placed on the straight edge of a board, and a perpendicular is ruled along the other side of the instrument, across the board. By sawi off the rough end, along this ruled line, the end of t board is made perpendicular to its side, or at right angl to it.

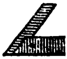


196. This instrument can also be used to make pe pendiculars on pasteboard, metal, or any thing that has flat surface and a straight edge.

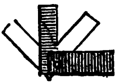
197. Sometimes the instrument is made thus, to allow of two right angles being made at the same time. It is called a T square.



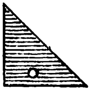
The instrument can be made with any angle



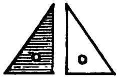
Or it can have a joint to allow it to open at any angle.

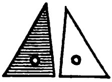


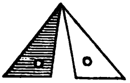
198. 3rd. With Triangle. A triangle made of wood or metal, which has one right angle, is the most convenient instrument for drawing perpendiculars and right angles on paper. One cut out of card will answer the purpose, if the angle be correct. Lines ruled against the short sides form a right angle.



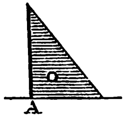
199. To find if the triangle's right angle is correct, rule a line and lay one short side against it. Then rule against the other short edge. Next, turn the triangle over on the second short edge and rule another line close to the first. If the instrument be correct, the two lines will be parallel. If incorrect, as at B and C, they will not be parallel.


A

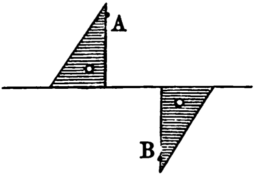

B


C

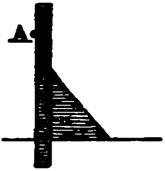
200. How to use the Triangle. To raise up a perpendicular from any point in a line, as from A: Place one of the shortest sides of the triangle upon the line, and rule a line against the other shortest side. The ruled line will be perpendicular to the given line, and the angle at A will be right angle.



201. To draw a perpendicular to a line from a point that lies out of the line. Let A be the point. Place one short side of the triangle upon the line, and slide the triangle along the line until the other short side touches the point: then rule a line from the

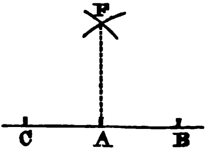


point A to the line. This ruled line is the perpendicular required. A perpendicular can be drawn from a point B below the line, in the same manner.



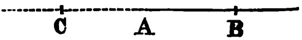
202. If the point A be too distant from the line for the triangle to reach it, lay a short side of the triangle on the line, and place a flat ruler against the other short side; then slide both along the line till the ruler touches the point. A line must then be drawn against the ruler from the point to the line.

203. 4th. With Compasses. To Raise a Perpendicular from any Point in a Line, as A. Take any point B in the line on one side of A; take another point C at the same distance on the other side of A; fix one leg of the compasses on B as a centre, and take any length with the other leg greater than AB, and draw an arc at F. Now take C as a centre, and, with the same distance in the compasses, draw another arc at F. Lastly, draw a line from F (the point where the two arcs cut) to A. This line AF is perpendicular to A.

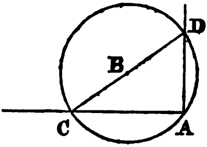


204. The two angles at A are right angles. If, therefore, we want to draw a right angle at any point in a line, as at A, we can do this in the same manner.

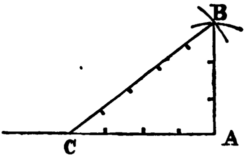
205. If the point A be at one end of the line, or very near the end, we must lengthen (produce) the line in the direction of C, and then take AC equal to AB, and do as before.



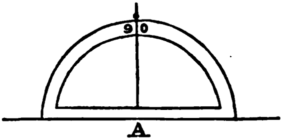
206. When the point A is near the edge of the paper, the line cannot be lengthened. In that case, take any point B above the line, near the point A. From the centre B, with a distance BA, draw a circle cutting the given line at C; join CB, and produce it till it meets the circumference at D; then join DA, and that line will be the perpendicular required.



207. Another Way. Take a distance of four inches (or parts) from A to C. From the centre A, with a distance of three inches (or parts), draw an arc B; from the centre C, with a length of five inches (or parts), draw another arc intersecting the first at B; join A with the point B, and the line AB will be the perpendicular required.

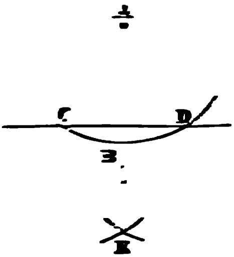


208. With a Protractor. To Raise a Perpendicular from a Point A in a Line. Place the lower edge of the protractor evenly on the line, so that the given point A shall touch the centre of the protractor; mark a point on the paper above the number 90, at the top of the protractor, and join the point with A. The joining line is a perpendicular raised on the given line from the given point A. For a description of the protractor see No. 262-9.



209. With Compasses. To Drop a Perpendicular from a Given Point A without (out of) a Line, CD.

Take A as a centre in the compasses, and take any distance as AB which extends to the other side of the line, and draw the arc CD.

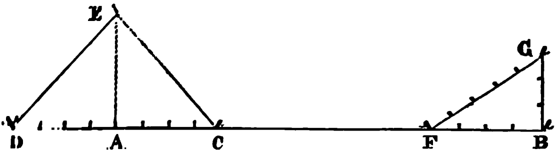


From C as a centre, with any distance greater than CB, draw an arc at E. From D as a centre, with the same distance, draw an arc at E, intersecting the first.

Join A to E. The line AE is perpendicular to the given line CD, and is drawn from the given point A without that line.

210. There are many other ways of drawing or marking out perpendiculars and right angles, according to the length of the lines and the circumstances under which they are made.

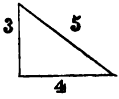
211. 7th. **Builders' Method with Cord and Pegs.** When a builder has fixed on a place for a house, he



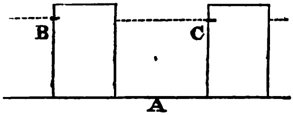
stretches a cord of the proper length from A to B for the front, say 16 yards, and sticks a peg in the two ends, A and B. He now wants to make a right angle at each of these points. First, for the angle at A, he takes a length of several yards from A to C, suppose four yards; he continues the line CA to D, also four yards from A, and sticks pegs in C and D, and makes the cord longer than CD, suppose twelve yards. Next, he stretches the cord towards E, so that there shall be exactly six yards on each side, namely, from E to D, and from E to C, and sticks in a peg at E. A line drawn from A to E is perpendicular to AB, and the angle at A is a right angle. When he has once got the right direction, AE, he can lengthen that line as much as may be required. If the building is to be square or oblong, he can find each of the four angles in this way.

212. But suppose that there is a wall close to B, which prevents him from producing A B in that direction: he must find another way of marking out the angle. He takes BF four yards, sticks pegs in B and F, and ties to these the ends of a cord eight yards long. He then extends this cord tightly, so as to make an angle, of which one leg GB shall be three yards long, and the other leg GF five yards long. He then sticks a peg at G and draws a line from G to B; which line is perpendicular to AB; and the angle at B is a right angle.

213. If a triangle has sides measuring 3, 4 and 5 inches, feet, or yards, it must be a right angled triangle. Also, if the sides measure 5, 12, and 13 inches &c. And a right angled triangle can be drawn with such threes of lines, as is shewn in No. 207.



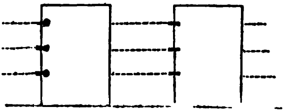
214. PARALLELS. TO DRAW A LINE PARALLEL TO ANOTHER. Suppose that we want to make a line parallel to the line A, through the point B.



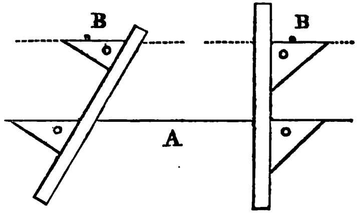
215. 1st. WITH A CARD. Lay one edge of the card on the line, so that another edge shall touch the point B.

Mark the card at the point B, then move the card towards the other end of the line, and dot the distance there, at C. A line between B and C is parallel to the line A. For distant parallels use a board instead of a card.

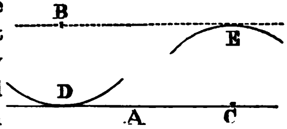
216. If there are several distances through which we want to make parallels to A, these distances can each be marked on the card, and then marked on the paper, near the other end of the line.



217. 2nd. WITH WOODEN TRIANGLE AND FLAT RULE. Let A be the line and B the point. Place a side of the triangle on the line, and place the flat rule against another side. Slide the triangle up against the rule until its upper side touches the point; and rule a line (see dotted line) against that side; which must be lengthened when necessary.

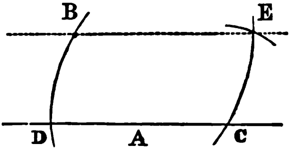


218. 3rd. WITH COMPASSES. From the point B as a centre, and with its distance from the line, draw an arc, D, just touching the line. From any point C near the opposite end of the line, as a centre, with the same distance, draw another arc, E. Then draw through B, a line touching the arc E, and that line will be the parallel required.

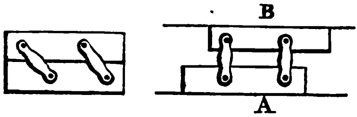


219. Distant parallels can also be drawn in this manner on the ground, with stakes at B and C; and cords.

220. 4th. With Compasses. In a part of the line A, the most distant from B, take any point C as a centre, and with CB as a radius, draw the arc BD. From B as a centre, with the same radius, draw the arc EC. From C as a centre, with the distance BD, draw a second arc at E. The line which joins B and E is parallel to the line A.



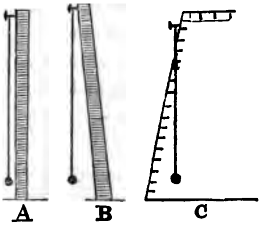
221. 5th. With Parallel Ruler. A parallel ruler is made of two flat rules joined by two equal cross pieces of metal jointed on, which allow one rule to move upwards while the other rule is kept fixed. The two rules always keep parallel to each other, by reason of the equal lengths of the two cross pieces.



Place the lower edge of the ruler on the line A, and, while keeping the lower rule steady, move the upper rule until its upper edge reaches the point B. Rule the parallel line through that point; lengthen it afterwards on either side, if necessary.

222. Vertical or Upright Lines and Objects. Vertical lines are also called *plumb* lines.

223. To find if a Post, or a Wall, or any other Object is quite Upright. Tie a bit of lead, or any small heavy object, to one end of a long string: then hold or fix the other end of the



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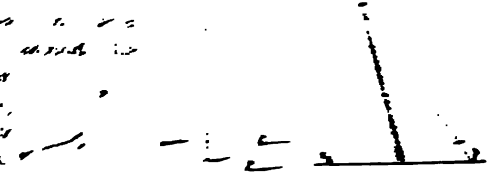
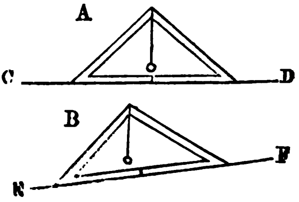


poles. Then measure three strings between the top of the pole and the pegs. If the strings are found to be of equal length, the pole is vertical; if not, it inclines towards the shortest string.

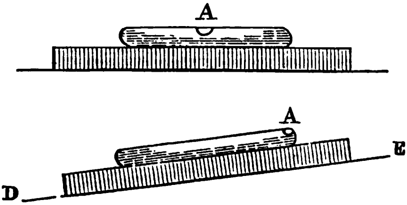
XIII. 1st. LEVELS. TO FIND IF A SURFACE IS LEVEL. (A surface that is perfectly flat, or horizontal, like the top of a pond, or table, or floor, is said to be level.) Place a plummet level on the surface. If the lead, which hangs vertically from the top by a string, comes to the middle of the instrument, as in diagram A, the surface CD is level. If the lead goes to the right or left of the centre, as in B, the surface EF is not level.



2nd. The plummet level is made [illegible]

bricklayers, carpenters, and many other trades, for laying stones, bricks, pavement, floors, and other objects, quite level.

227. 2nd. Spirit Level. Another instrument which is much used in taking levels, is a long narrow glass bottle, nearly filled with spirits of wine, a single bubble of Air (A) being left. When the glass is on a perfect level, the bubble keeps in the centre; but if the surface is not level, as DE, the bubble runs to the upper end.



228. To Bisect a Line, that is, to divide it into two equal parts.

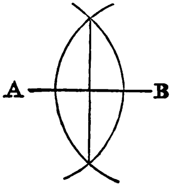
1st. With a Slip of Paper. Take the exact length of the line, on a narrow slip of paper; cut off that length, and fold the slip in two exactly.

229. 2nd. With a String or Cord. Measure the length on a string or Cord, and double the string or cord into two equally. This way is useful for long lines, when we have not a measure.

230. 3rd. With a Measure. Measure the length of the line in inches, or feet, &c., and take half the number of inches, or feet, &c.

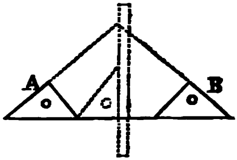
231. 4th. With Compasses. Make trials with the compasses, till the exact half of the line is got.

232. 5th. With Compasses. From one end A of the line as a centre, with any distance greater than half the line, draw an arc to some distance on each

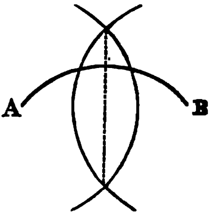


side the line. From B, with the same distance, draw a second arc, intersecting the first above and below the line. Join the two points where the arcs intersect. The joining line divides AB into two equal parts.

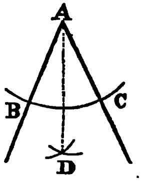
233. 6th. With Wooden Triangle. First, place the longest side of the triangle on the left end of the line, and trace an outer inclined line A, which makes an angle with the line. Next, place the same wooden angle on the other end of the line, and draw a second inclined line B. Continue the two inclined lines till they intersect. Lastly, place one of the short sides of the wooden right angle on the line, and slide it towards the middle till the other short side (or the edge of a flat rule placed along it) touches the point of intersection, and rule the perpendicular. This bisects the given line.



234. To Bisect an Arc. The arc AB can be divided into two equal parts, with compasses, as the line was, in the 4th and 5th ways of the last number.

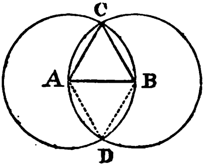


235. To Bisect an Angle, or Divide an Angle into Two Equal Angles. From the centre A, with any length shorter than a leg of the angle, draw the arc BC. From the point B as a centre, with any distance greater than half of BC, draw an arc at D. From



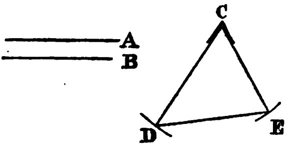
C as a centre, with the same distance, draw another arc at D. Join AD; this line divides the angle at A into two equal angles.

236. An Equilateral Triangle drawn on a Given Line, AB. From A as a centre, with the length of the line AB, draw a circle. From B as a centre, with the same distance BA, draw another circle. Join AC and BC. The triangle ABC is equilateral.



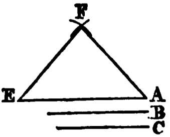
237. A Rhomb Drawn. Join the points AD and BD. A second equilateral triangle is then drawn; and the two triangles, when joined, form a rhomb.

238. To Draw a Triangle when Two Sides, A and B, and The Angle C to be Contained by Them, are given. From C as a centre, draw an arc D, with the length of A. From the same centre with the length of B, draw another arc, E. Produce, or lengthen, the legs of the angle till they reach D and E, and draw the base line DE, which completes the triangle.



239. To draw a Triangle with three Given Lines A, B and C for Sides. But any two of the lines must be longer that the third (No. 82).

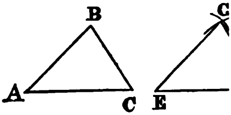
Take A for base. From the right end as a centre, with the length of B, draw an arc at F. From E as a centre, with the length of C, draw another arc at F. Draw lines



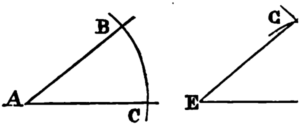
from E and A to F, where the arcs cut. The triangle has three sides equal to the three given lines.

If these sides are sticks or poles, they can be together at once in a triangle.

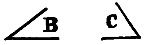
240. To Draw an Angle Equal to the Given An Take any points B and C in the legs of the given angle, and join them. Draw a line EF equal to AC, and make a triangle GEF with sides equal and similarly placed to those in the tr BAC. (See last No.) The angle E is equal to A.

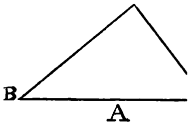


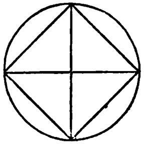
241. Another Way. From the centre A draw BC, intersecting the legs of the angle. Take any line EF as base, and from E as a centre, at the distance of AB or AC, draw the arc GF. From F as a with the distance CB, draw another intersecting arc and join EG. The angle at E is equal to A.



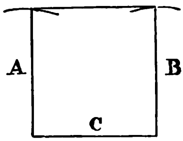
242. To Draw a Triangle when Two Angle the Side between them are Given. Make ang the ends of the given line A, equal to the given angles B and C (as in the last No). Produce the two oblique lines until they meet in a third angle or apex.







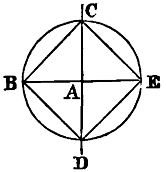
243. A **Square** made. 1st. **Folded in Paper.** Cut out a large paper circle. (No. 172.) Fold one diameter: and then another perpendicular to the first, by folding one-half of the first on the other half. Then fold down and cut off the four curved segments.



244. 2nd. **With Wooden Triangle.** Draw a base line C, of the length wanted. At each of its ends erect a perpendicular, A and B, and make them equal to the base; and join their upper ends.

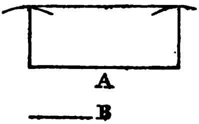
245. 3rd. **With Compasses.** First way as with paper-folding, above; namely, draw a circle and two diameters at right angles; and join the ends of the diameters.

Second way. As with triangle above, viz: erect two perpendiculars at the ends of a base (No 200), take each of those ends as a centre, with the length of the base as radius, and cut off an equal length from A and B, by arcs. Join the upper ends of these two equal lines.



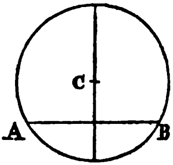
246. 4th. Third way. **When the Diagonal only is Known, to Draw a Square.** Bisect the given diagonal BE by a line at right angles to it, (No. 228—233). From the centre A, with the distance AB, draw a circle cutting the perpendicular at C and D, and join these points with B and E. BCED will be a square, of which BE and CD will be diagonals.

247. An Oblong or Rectangle made with long and short sides equal to the two given lines A and B. Draw perpendiculars from the ends of A: make them each equal to B; and join their upper ends. This may be done with the wooden triangle or the compasses.



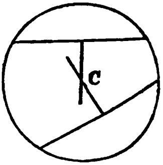
248. A Circle Drawn when the Diameter only is Known. Bisect the diameter, and draw the circle from the centre, with half the diameter as radius.

249. To Find the Centre of a Circle. Draw any chord across the circle, as AB; bisect it, and raise a perpendicular, which must be produced to the opposite points of the circumference. Bisect the perpendicular at C. C is the centre of the circle.

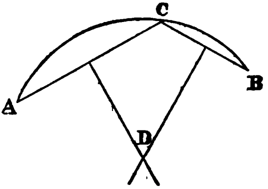


250. Another Way. Draw any two chords, not parallel; bisect them; and from the bisecting points raise perpendiculars. The point C, at which these perpendiculars meet, is the centre of the circle.

251. By Folding Paper. When a circle is cut out of paper, its centre can be found in either of these ways by folding chords, and then finding the perpendiculars by folding one half of each chord on the other half.



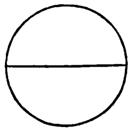
252. To Find the Circle of which a Given Arc is a Part. Divide the arc into any two parts, AC, CB. Draw chords to these two arcs, and raise perpendiculars on their middle points. The point D, where these perpendiculars cut, is the centre of the circle; and the circle can be drawn with a distance of DA, or DB, or DC.

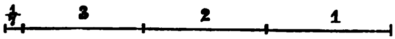


253. To Draw a Circle through any Three Points which are not in the same Straight Line. Let A, B, and C be the three points. See last diagram. Join AC, and BC; bisect the lines AC and BC: and from their middle points, draw perpendiculars intersecting in D. The circle can be drawn from the centre D, with a distance of DA, or DB, or DC.

254. A Circle made Equal to a Given Square. Divide a side of the square into eight equal parts. Draw a line equal to nine such parts. This line is the diameter of the circle. Bisect it and draw the circle. This method is nearly correct, but not quite so.

255. To Find the Length of the Circumference of a Circle. Measure the length of the diameter of the circle. Draw a straight line; and mark off, upon it, three times the length of the diameter, and one-seventh of the length of the diameter. If the diameter measure one foot, the circumference will be three feet and one seventh of a foot; or 3 feet $1\frac{3}{4}$ inches





Length of Circumference.

nearly. If the circumference be a yard (36 inches), the diameter must be about 11½ inches.

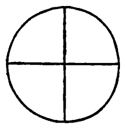
256. When the length of the diameter is known, multiply it by 22, and divide the product by 7; this gives the length of the circumference; it amounts to the same thing as multiplying the diameter by 3 and 1-seventh, and avoids the fraction 1-seventh. Example,—length of diameter 14. A more correct result would be obtained in this and the following problem, by using 355 instead of 22, and 113 instead of 7.

$$\begin{array}{r} 14\\ 22\\ \hline 28\\ 28\\ \hline 7)308\\ \hline 44\\ \hline\hline \end{array}$$

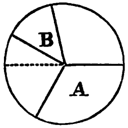
257. To Find the Length of the Diameter of a Circle when the Length of the Circumference is Known. Multiply the circumference by 7, and divide the product by 22. Example. If length of circumference be 44; 44 × 7 = 308; 308 divided by 22, is 14, which is the length of the diameter, and 7 is the length of the radius. If the circumference be 110; 110 × 7 = 770; and 770 divided by 22, is 35, the length of the diameter.

258. Measurement of Angles. It is necessary that we shall be able to measure and write down the size of any angle, that another person shall be able to draw the angle from our description only.

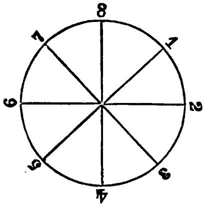
259. A right angle is contained by a horizontal and a perpendicular line. If we draw a circle, and make a horizontal and a perpendicular diameter in it, there will be four right angles at the centre, and each right angle will have a quarter of the circumference opposite it. A right angle is therefore said to be measured by a quarter of the circumference of



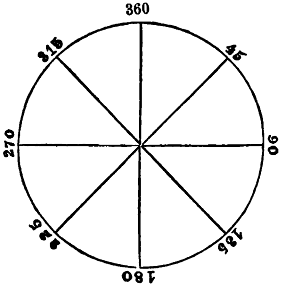
a circle. If an angle be obtuse, it must have more than a quarter opposite it, as A: if acute, it will have less than a quarter, as B.



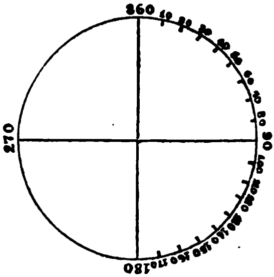
260. If the circumference of a circle be divided into eight equal parts, each part will have half a right angle opposite it: and two such parts will be opposite one right angle; and three parts opposite an obtuse angle that is equal to a right angle and a half.



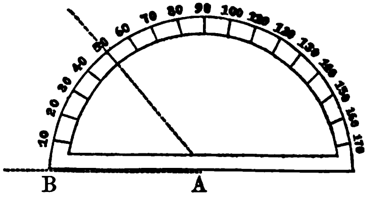
261. The size of angles is measured, and described or written, by the number of parts of the circumference that an angle at the centre has opposite it. But since we can measure very few angles if the parts are so large as eighths of a circle, the circumference is divided into 360 parts, which are called degrees (not parts); 90 degrees are opposite to a right angle: 45 degrees are opposite to an acute angle that is half a right angle. 135 degrees are opposite an obtuse angle equal to a right angle and a half. A small ° is placed above a number, thus, 90°, 45°, to signify that the number means degrees. When a circle is sufficiently large to have every degree marked upon it, angles of a great many



sizes can be measured by placing the angle on the centre, and noticing how much of the circumference it covers. 89 acute angles can be measured from 1° to 89°; and as many obtuse angles larger than 90° and less than 180°; for no single angle can be quite so large as two right angles, or 180°.

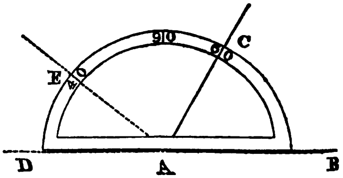


262. A brass semicircle, or rim, called a protractor, is used for measuring angles. It is divided into 180 parts, though, usually, there is only room on it for marking the figures for every tenth number.



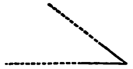
263. To Measure an Angle with the Protractor, place the angular point at A, which is the centre of the circle, and one leg of the angle on the diameter at AB (see dotted line), the other leg (dotted) goes through the 50th degree. This angle therefore measures 50°. If there had been a right angle, it would have gone through 90°.

264. The degrees are numbered in two ways on the protractor, viz: from the bottom of the right side to the left; and *from* the bottom of



the left to the right. If the point of the angle lie towards the left, the degrees are counted on the right side of the protractor, viz BC. If the point lie towards the right, the degrees are counted on the left side, D to E.

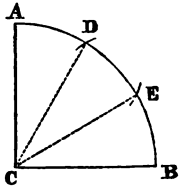
265. If the inclined (or upper) leg of the angle be not long enough to reach the curved rim of the protractor, it must be produced until it does go beyond the curved rim.



266. For some purposes it is necessary that angles should be measured to a much greater degree of nicety. Each degree is then divided into 60 parts, which are called minutes, and marked so 5′. Thus 30° 5′ signifies that the angle measures 30 degrees and 5 minutes, or 5-sixtieths of another degree.

267. In some cases it is necessary to measure still smaller angles. The minute is then divided into 60 parts called seconds, and marked so ″. 7″ means 7 seconds or 7-sixtieths of a minute. 30° 5′ 7″ signifies that the angle measures 7 seconds more than 30° 5′.

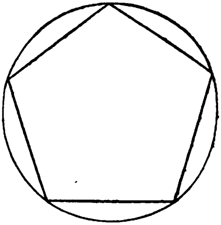
268. To Divide a Quadrant into Three Parts of 30° each. Let ABC be the quadrant. From a centre B, with a radius of BC, draw an arc D. From a centre A, with the same radius, draw an arc E. The arc AB is divided, at D and E, into three equal parts of 30° each. Each arc would therefore be opposite to, and measure, one third of a right angle. If each of these arcs, viz: AD, DE and EB, be divided by the compasses into three parts, the quadrant will be divided into nine parts of ten degrees each.



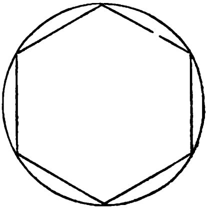
269. To Make a Protractor for Measuring Angles. Draw a large semicircle on stiff paper or card. Divide it by

a perpendicular from the centre, into two quadrants. Divide each quadrant into nine parts, as above. Divide each of these parts into ten parts by trials with the compasses. The semicircle or protractor is then divided into degrees. Place a number to every tenth degree, viz: 10, 20, 30 &c., to 180, from right to left, and also from left to right. Then cut out the protractor for use. (See diagram at No.262).

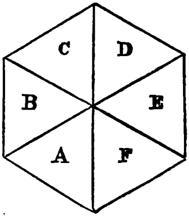
270. PENTAGON. TO DRAW AN EQUILATERAL AND EQUIANGULAR PENTAGON. Draw a circle; divide the circumference by trials with the compasses, into five equal arcs. Draw straight lines from each point to the two points nearest it.



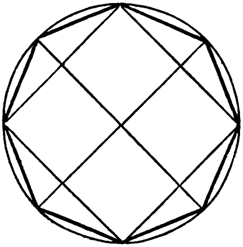
271. HEXAGON. TO DRAW AN EQUILATERAL AND EQUIANGULAR HEXAGON. 1st way. Take the length of the radius of a circle in the compasses, and mark off six points in the circumference. Join each point to the two nearest to it.



272. 2nd way. BY TRACING. Mark a central point: take an equilateral triangle cut out of thick paper (No 236). Place one of its angles in the centre, and make a tracing round the triangle, viz., A. In the same way trace the triangles B, C, D, E and F round the central point. The hexagon *is* formed by the bases of these six *triangles.*

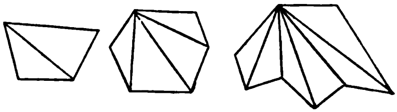


273. Octagon. To Draw an Equilateral and Equiangular Octagon. Draw a square in a circle. (No. 246.) Bisect each side of the square, (No. 231—3) and draw perpendiculars through the middle points to the circumference. Join the end of each line to the ends of the two next lines.



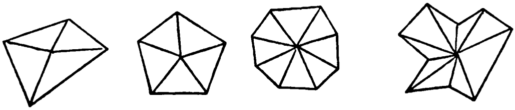
274. Figures Divided into Triangles. To Divide any Figure that has more than Three Sides, into Triangles.

1st way. Draw diagonals from any one angle to the opposite angle or angles.



The number of the triangles is two less than the number of the sides of the figure.

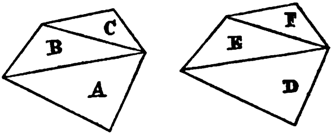
275. 2nd way. Make a point any where within the figure, and draw lines from it to each angle. By this mode, the number of the triangles is equal to the number of the sides of a figure.



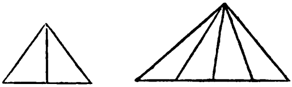
276. To Draw any Right-lined Figure Similar and Equal to Another. If the given figure be a triangle, draw

another with corresponding sides as long as those of the first. (See No. 239.)

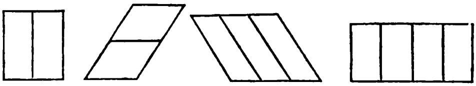
277. If the given figure have more than three sides, divide it into triangles by means of diagonals. Make a base line equal to the base of A, at D; and complete the triangle D equal to A. (No. 239.) The upper side of D is equal to the base of B. Draw on that upper side the triangle E equal to B. Complete the pentagon by drawing the triangle F equal to C.



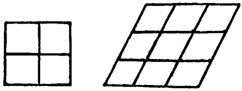
278. To Divide a Triangle into any Number of Equal Triangles. Divide the base into the required number of equal parts, and draw lines from the apex to the points of division. These triangles cover equal spaces, but they are not similar. (No. 131.)



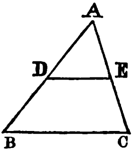
279. To Divide a Parallelogram (*i. e.* a Square, Oblong, Rhomb, or Rhomboid,) into any Number of Equal Parallelograms. Divide any side into the number of equal parts required, and draw lines to the opposite side parallel to the two remaining sides. The smaller figures are similar to one another as well as equal.



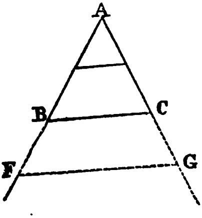
280. If two adjoining sides be divided into the same number of parts, and parallels be drawn, as before, the smaller figures will be equal to one another, and similar both to one another and to the whole figure.



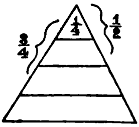
281. To Draw a Triangle of any size Similar to any other Triangle. If the required triangle is to be smaller than the given triangle ABC, suppose with sides half the length, bisect AB, and from the dividing point, draw a line within ABC, parallel to its base BC. The base DE is as much shorter than BC as AD is shorter than AB, or AE than AC. And each angle of the smaller is equal to the corresponding angle of the larger triangle.



282. If the required similar triangle is to be larger than the given triangle ABC, suppose a half larger, produce the sides AB AC; take BF equal to half AB, and through F draw FG parallel to BC. If the side AB measures six inches, AF must measure nine inches; and FG will be a half longer than BC, and AG a half longer than AC. The corresponding angles of the two triangles will also match.

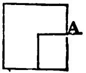


283. When the sides of one figure are half the length of those of a similar figure, the smaller figure is on half the scale of the larger. If the sides of the smaller figure are a quarter or three quarters of the larger, the smaller is on a quarter or three quarters the scale of the larger. In such

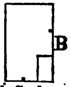


cases the larger figure is on twice, or four times, or one and a third times the scale of the smaller.

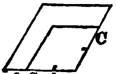
284. To Draw a Parallelogram Similar to a Given Parallelogram, and with Sides of any Comparative Length, as Half as Long, Twice as Long, One-third as Long, &c. If the sides of the required figure are to be the shortest (as ½, or ⅓, or ⅔, or ¾), take the part required on two adjacent sides, and draw parallels from these points to the two opposite sides. See diagrams A, B, C and D.



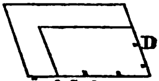
½ *Scale.*



⅓ *Scale.*

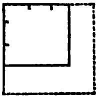


⅔ *Scale.*

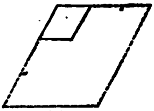


¾ *Scale.*

285. If the figure is wanted larger than the given figure, prolong two adjacent sides of the given figure (as dotted) to the length required, and draw parallels to the opposite sides. See diagrams E and F.

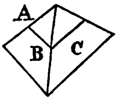


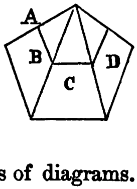
E

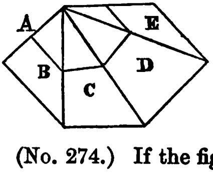


F

286. Similar Polygons Drawn to a Larger or Smaller Scale. If the similar figure is required to be smaller than the given figure, divide the given figure into triangles, by means of diagrams. (No. 274.) If the figure wanted be of half the scale, bisect the side of an outermost

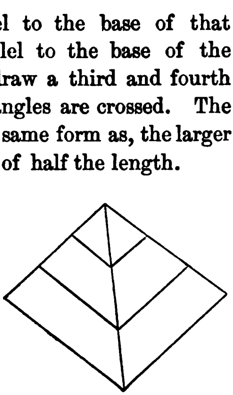




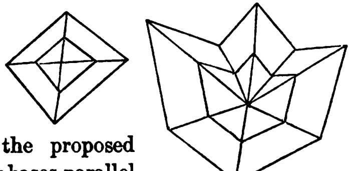


triangle A, and draw AB parallel to the base of that triangle. From B draw BC parallel to the base of the adjoining triangle. If necessary, draw a third and fourth parallel CD, DE, until all the triangles are crossed. The inner figure is similar to, or of the same form as, the larger figure, and has corresponding sides of half the length.

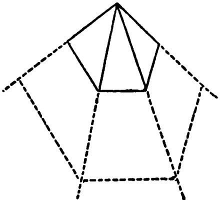
287. Similar figures of a quarter scale, one-third scale, three-quarter scale, and any other scale, could be drawn in the same manner, by taking a quarter, a third, or three quarters, &c., of the first or outermost line.



288. Sometimes it is more convenient to divide the figure into triangles by lines from a central point. (No. 275.) Divide, then, either side of any one of the triangles, according to the proposed scale, and draw shorter bases parallel to the longer bases, as before. The inner figure is similar to the outer or larger one, and in both the diagrams, is of half the scale of the original figure.



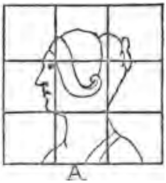
289. If the similar figure is required to be of a larger scale than the given figure, draw diagonals from one angle, and produce the two outer sides, and the diagonals. (See dotted lines.) Make any outer side or diagonal double, or as much larger as is

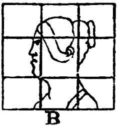


wanted, and draw long bases parallel to the short ones of the original triangles. Each of the large dotted figures is similar to the corresponding smaller one, and is drawn to double the scale.

290. Figures of three times the scale, or four times, or one-and-a-half scale, or of any scale larger than the original figures, can be drawn in the same manner.

291. Similar figures can also be drawn to any scale by dividing A, the plane of the original, into a number of equal squares, and then dividing the paper for the copy, B, into the same number of squares. Notice what parts of the original are in each square, and draw corresponding lines in the same squares of the copy.

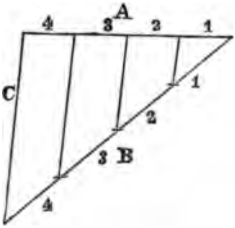

A


B

When the original is a map or drawing with many small parts, it must be divided into many squares, and these must be numbered, both in the original and copy, to prevent mistakes. When lines cannot be drawn on the original, a net-work of threads may be stretched over it. Draughtsmen frame net-work of different sizes for this purpose.

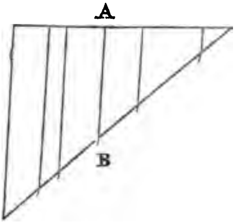
These methods are of great use in drawing plans, maps, and charts, of a smaller or larger size than the copy, and are constantly employed by artists, engravers, map-makers and engravers, surveyors and engineers.

292. To Divide a Straight Line A into any Number of Equal Parts. If the line A is to be divided into four



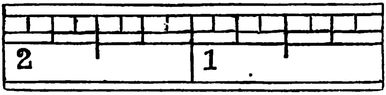
equal parts, let another line B, of any convenient length, be drawn, making any angle with A. Along this line measure four equal lengths, and mark the end of each, 1, 2, 3, 4. Join the last mark with the distant end of A by the line C, and draw parallels to this line from the points marked 1, 2, 3. The line A is divided by these parallels into four equal parts.

Each of the small triangles in this figure is similar to the other two, and to the largest triangle.



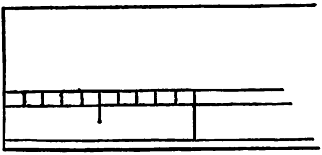
293. If it be required to divide the line A into any number of unequal parts, corresponding to the parts of B, the division can be performed in the same manner.

294. To Measure on a Scale. A scale is a measure divided into inches and parts of an inch. On some measures the inches are divided into halves, quarters and eighths, in this manner, as can be seen on a carpenter's folding rule, where the inches are separated by lines across the rule, the halves of inches are separated by shorter lines; the quarter inches by shorter lines still: and the eighths of inches by

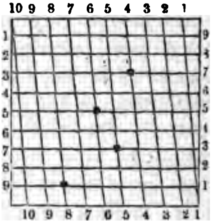


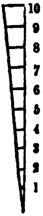
the shortest lines of all. The shortest distance that can be measured on this rule is the eighth of an inch.

295. The flat scale put in boxes of instruments, has the inches divided into tenths, in this manner; not into eighths. The inches and half inches are divided in the same way as on the carpenter's rule. All the other divisions marked with very short lines, are tenths of an inch; for the quarters and eighths cannot be shewn by this manner of dividing. The tenth of an inch is the shortest distance that can be measured on this rule.



296. Diagonal Scale. But a part of this rule is usually divided to allow of much smaller lengths being measured. The top and bottom of a square inch are divided into ten parts. These parts are joined by inclined lines. The two sides are divided also into ten parts; but these parts are joined by horizontal lines. The distance between any two adjoining inclined lines is one tenth of an inch. But the distances between the first inclined line and the side of the square, vary in length from one tenth of an inch at top to one hundredth of an inch at bottom. The same is the case with the last inclined line and the left side of the square, except that the size of the parts diminishes upwards.



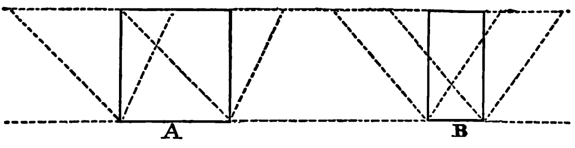


297. By this addition to the scale or measure, lengths of inches, tenths of an inch and hundredths of an inch can be measured. Thus, the highest dot in the square, measured from the right perpendicular, is $\frac{3}{10}$ths, $\frac{7}{100}$ths of an inch long. The next dot is $\frac{5}{10}$ths, $\frac{5}{100}$ths. The third dot is $\frac{4}{10}$ths, $\frac{3}{100}$ ths. The lowest dot is $\frac{7}{10}$ths and $\frac{1}{100}$th of an inch.

298. When the flat scale is small, a half-inch is taken for division in this manner, instead of an inch; and also a quarter of an inch. The parts measured on the former are $\frac{1}{200}$th of an inch, $\frac{2}{200}$ths, $\frac{3}{200}$ths, and so on to $\frac{10}{200}$ths, which is $\frac{1}{20}$th of an inch, being the base of the whole triangle formed by the inclined line and the perpendicular. On the quarter-inch scale, the parts are $\frac{1}{400}$th of an inch, $\frac{2}{400}$ths, and so on to the base of the whole triangle, which is $\frac{10}{400}$ths, the same as $\frac{1}{40}$th part of an inch.

299. A drawing or plan can be made to any scale we chose: that is, we can take any other length than an inch, such as this ——— and divide it into tenths; and also make a square scale, like that above, to divide it into hundredth parts.

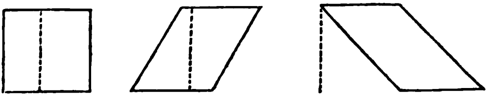
300. TO DRAW A PARALLELOGRAM EQUAL IN AREA TO ANY OTHER OF A DIFFERENT SHAPE. All parallelograms, on the same base and between the same parallels, have equal



areas; that is, they cover an equal surface. They are also equal if they are on equal bases and between the same parallels. Or if they are on the same bases, or on equal bases, and of the same altitude, or perpendicular height. (No. 68.)

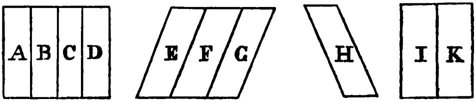
The square A covers as much space as either of the two dotted rhomboids, inclining to the right and left. The oblong B is equal to either of the two dotted rhomboids on the same base.

These three figures are also equal: being on equal bases,



and of the same perpendicular height, which is shewn by the equal dotted perpendiculars.

301. A parallelogram on half, or a quarter or three quarters the base of another, and between the same parallels

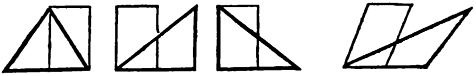


(or of the same altitude) has half, or a quarter, or three quarters the area of the other. The narrow figure A is a quarter of the large oblong. The figure made up of ABC, is three quarters of the whole oblong. The rhomboid E has one-third of the surface of the rhomboid EFG. The rhomboid H is half of the oblong IK. And the oblong I or K is also half of the whole oblong.

302. If a parallelogram be on the same base as another, but only of half, or a quarter, or three quarters the altitude, it will cover only half, or a quarter, or three quarters of the surface, as is seen by turning several of the above diagrams sideways.

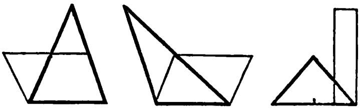
303. A Parallelogram Drawn Equal to a Given Triangle. A parallelogram has double the area of a triangle on the same base (or on equal bases), and between the

same parallels (or of the same altitude). Bisect the base of the triangle by any line, and draw a line through the apex (or top angle) parallel to the base, to meet the bisecting line. Draw another line parallel to the bisecting line, which will complete the parallelogram.

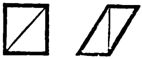


We can also draw the equal parallelogram on any line equal to half the base of the triangle, provided we keep the altitude of the two figures equal.

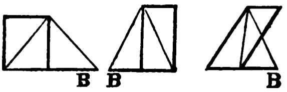
304. If the parallelogram be drawn on the whole base of the triangle, it must have only half the altitude of the triangle. If it be drawn on a quarter of the base, it must have twice the altitude.



305. A TRIANGLE DRAWN EQUAL TO ANY PARALLELOGRAM. 1st. Draw a diagonal across the parallelogram: the parallelogram is then divided into two equal triangles.

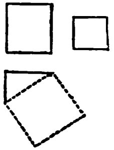


2nd. Double the base, and join the farthest end B to the apex of the first triangle, a second triangle is then formed, equal in area, to that on the first base. These triangles together form one larger triangle, which is equal in area to the parallelogram.

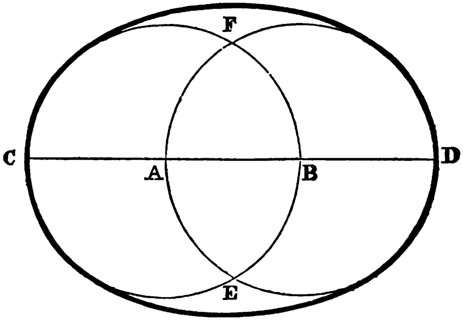


Or an equal triangle could be drawn on any line double of the parallelogram's base, with the same altitude.

306. **To Make a Square Equal to any two Other Squares.** Make a right angle with one side of each of the two smaller squares. Join the distant ends of the sides; and on this third line draw a square, which will cover an area equal to that of both the smaller squares.



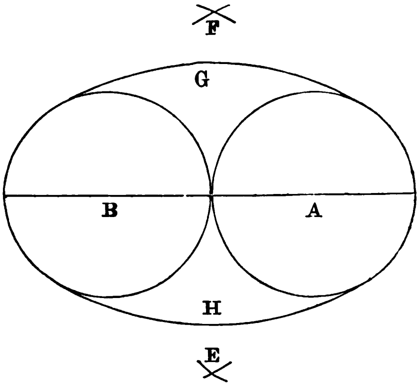
307. **Oval. To Draw an Oval.** Divide a line into three equal parts. Draw a circle from the centre A with the distance AB, and another circle from the centre B, and with the same distance.



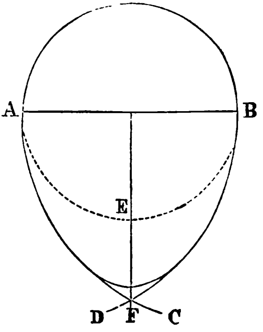
From the point of intersection E, and a distance of two parts, draw an arc above F to join the circumferences of the two circles. From the point F, with the same distance, draw an arc below E, which completes the oval.

308. **A Long Oval Drawn.** Divide the line into four *parts.* Draw two circles from centres A and B, with radius

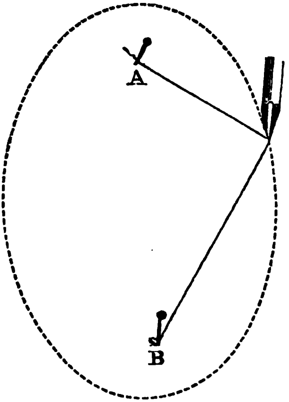
of one part. Then take A and B as centres, with a distance of two parts, and make arcs cutting at E and F. From E and F as centres, with a distance of three parts, draw the arcs G and H, joining the circumference of the circles.



309. **An Egg-shaped Oval Drawn.** Draw a semi-circle; draw a perpendicular from the middle of its diameter. From the ends of the diameter A and B, with the length of the diameter, draw arcs BD, AC. With centre E, and four-fifths the distance of EF, draw a small arc to unite the two larger arcs, and complete the oval.

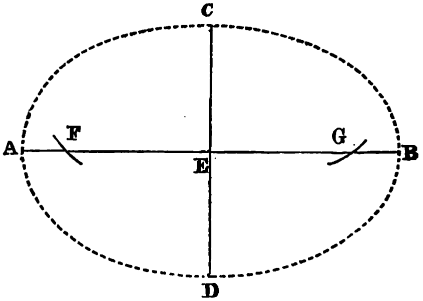


F



310. **An Ellipse Drawn.** Stick two pins A and B upright into paper on a board. Tie one end of a stout thread to each pin, but let the thread be very loose. Cut a notch as near as possible to the point of a pencil; put the thread into that notch, and mark round with the pencil, always keeping it upright and the thread stretched. The less loose the thread is, the narrower the ellipse will be. (See No. 124.)

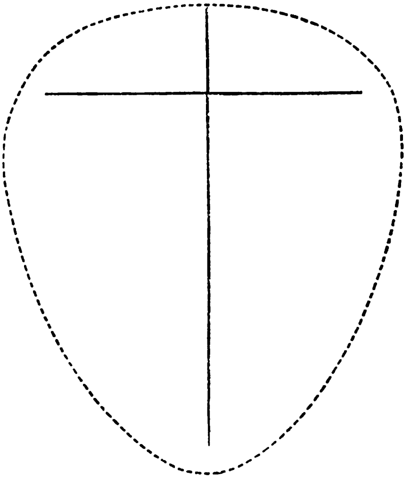
Large ellipses, such as garden beds, can be drawn on the ground in this manner, with a cord and pegs.



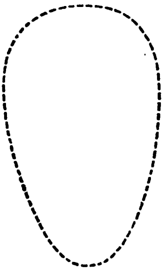
311. **An Ellipse Drawn of a Given Length and Breadth.** Supposing that an ellipse is wanted to measur

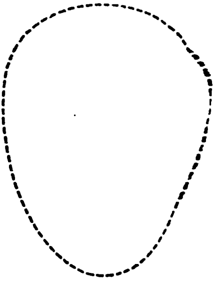
ıches the longest way, and four inches the shortest Draw a horizontal line AB six inches long, bisect it perpendicular line CD four inches long. From the e C, with the distance EA, draw arcs F and G on the)ntal line. Stick pins upright in F, G, and C, and tie)ng thread tightly round them. Then take out the t C, and draw the ellipse by keeping the thread ded with the point of a pencil, as was done in 10.

. A Compound Ellipse Drawn. Draw a perpendi- bisecting a horizontal line, but make the lower part : perpen- r much · than the part of Fix four or nails ıt into ıds of the and tie a · thread ŗ round Take the top Extend read with)oint of ıcil, and a curved ll round, as was done in drawing the ellipse.



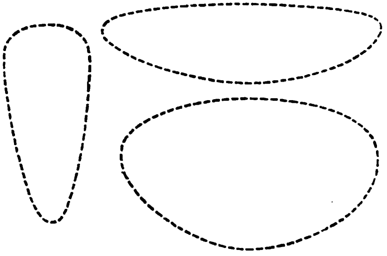
. A great variety of beautiful forms can be made by ıg the length of the perpendicular in comparison with the length of the horizontal line. Also, by making the lower part of the perpendicular longer or shorter; but it should never be less than twice as long as the upper part.





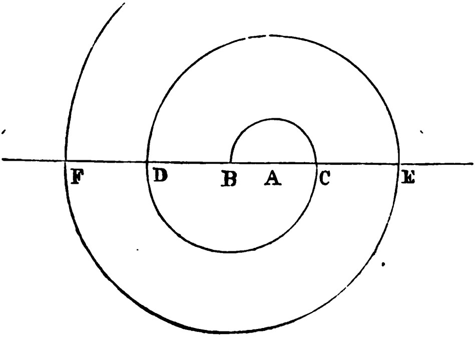
314. This is a much more perfect figure than the egg-shaped oval, and can have an infinitely greater variety of forms. It is of much use in manufactures and ornaments.

New forms of tea-pots, cups, jugs, vases, urns, mouldings, and ornaments of many kinds can be designed by cutting out these figures, and tracing the whole, or part, of one or more of them for the object to be designed. (See Hay's *First Principles of Symmetrical Beauty*, parts III and IV, for many varieties.)



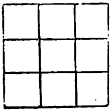
315. A SPIRAL LINE DRAWN. Draw a straight line, and divide it into a number of equal parts. Take a middle part *BC* and bisect it. From its centre A, with a length AB,

draw a semicircle. From the centre B, with the length BC, draw a second semicircle, joining one end of the first at C. From the centre A, at a distance AD, draw a third semicircle, joining the second at D. From the centre B, at the distance of BE, draw a fourth semicircle, joining the third at E. Draw a fifth semicircle from A, and a sixth from B, and keep on changing the centres, and encreasing distances, until a sufficient length of spiral is drawn.

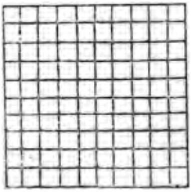


SQUARE MEASURE.

316. Square. To Find How Much Space a Square Covers. This is called its square measure, because we find how many smaller squares of a known size, it is equal to; such as square inches, or square feet &c. This is also called the area of the square. Multiply the length of one side by itself. A square with a side 3 inches long, (called a 3-inch square) measures 3 times 3 or 9 square



inches, or squares with one inch sides. A yard square measures nine square feet.



A square ten inches in the side (or a ten-inch square) covers an area or surface of 100 square inches; for 10 times 10 are 100.

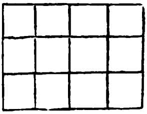
A foot square measures 144 square inches, because a foot is 12 inches, and 12 × 12 = 144.

If the side of a square is not an exact number of feet or yards &c., as, for example, two yards and two feet, or two feet and five inches: two yards two feet are eight feet; 8 × 8 = 64 square feet. 64 divided by 9 are 7 square yards, 1 square foot, the area of the first square. 2 feet 5 inches are 29 inches: 29 × 29 = 841 square inches; 841 divided by 144 are 5 square feet 121 square inches, the area of the second square.

If a surface covers one or more square yards, we can find how many square feet it covers by multiplying the yards by 9. If it cover 4 square yards, it must cover 4 × 9 or 36 square feet. We can also find how many square inches it covers by multiplying the square yards by 1,296, for 36 × 36 = 1,296.

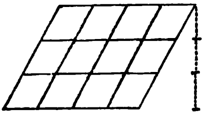
If a surface contains one or more square feet, we can find the square inches it covers by multiplying the square feet by 144; because 12 inches multiplied by 12 inches = 144. A table containing 3 square feet of board has a surface of 3 × 144 or 432 square inches.

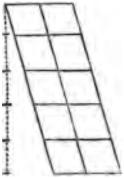
317. OBLONG OR RECTANGLE. TO FIND ITS AREA OR SURFACE. Multiply a long side by a short side. An oblong 3 inches by 4, that is, with short sides 3 inches each, and long sides 4 inches, measures 3 *times 4* or 12 square inches.



A rectangle 6 inches by 8 inches measures 6 × 8, or 48 square inches.

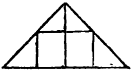
318. Rhomb and Rhomboid: to Find Their Area. Multiply the length of the base by the altitude or perpendicular height. A Rhomb of four feet base and three feet altitude, has an area of 4 × 3 or 12 square feet. A rhomboid of 2 feet base and 5 feet altitude has an area of 10 square feet.



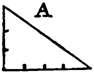


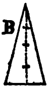
319. Triangle: To Find its Area; or the Space it Covers; or its Square Measure. Multiply the length of the base by the altitude (or perpendicular height) and divide the product by 2.

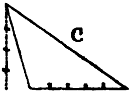
If the base measure 4 inches, and the altitude be 2 inches, twice 4 are 8, and half 8 are 4. The surface or area of this triangle measures 4 square inches.

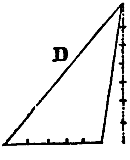


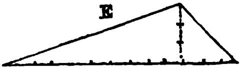
In these diagrams the altitude is dotted:







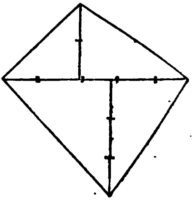




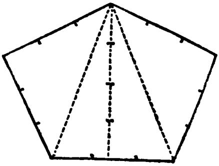
A measures the half of 4 times 3 which is half 12 or 6 sq. ins.
B ,, ,, 2 ,, 4 ,, 8 ,, 4 ,,
C ,, ,, 5 ,, 4 ,, 20 ,, 10 ,,
D ,, ,, 5 ,, 7 ,, 35 ,, 17½ ,,
E ,, ,, 12 ,, 3 ,, 36 ,, 18 ,,

320. POLYGONS. TO FIND THE AREA OF ANY FIGURE WITH MORE THAN THREE SIDES. Divide it into triangles, (as in No. 274): measure each triangle (No. 319), and add their measures together.

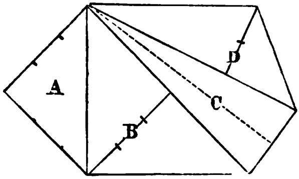
321. This TRAPEZIUM is composed of two triangles: one of which measures 5 in base, and 2 in altitude; and its area is the half of 10 square inches or feet &c. The second triangle measures 5 of base by 3 of altitude. Its area is the half of 15 or 7½. Total area of trapezium 12½ square inches.



322. This PENTAGON is divided into three triangles. The middle one has a base of 3, and altitude of 4, and measures 6 square inches &c. Each of the side triangles has a base and altitude of 3, and measures 4½ square inches &c., together 9 square inches: 9 and 6 are 15 square inches, the square measure of the whole pentagon.



323. This HEXAGON is divided into 4 triangles

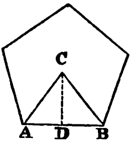


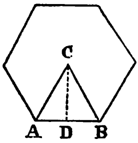
A	base	3,	altitude	3,	area	$\frac{3 \times 3}{2} = 4\frac{1}{2}$
B	,,	6	,,	3	,,	$\frac{3 \times 6}{2} = 9$
C	,,	2	,,	6	,,	$\frac{2 \times 6}{2} = 6$
D	,,	6	nearly	2	,,	$\frac{6 \times 2}{2} = 6$

Total area or square measure, $25\frac{1}{2}$ square inches, or feet &c.

324. TO FIND THE AREA OF ANY REGULAR POLYGON, (THAT IS, OF ONE WITH EQUAL SIDES AND ANGLES) WHEN THE LENGTH OF ONE SIDE AND THE CENTRE ARE KNOWN.

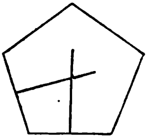
Multiply together the side AB, and the perpendicular to the centre, and also the number of sides; and take half the product. If AB be 10, and CD 9, and there be six sides, then $10 \times 9 = 90$, and $90 \times 6 = 540$; and $\frac{540}{2} = 270$.





If we do not know the centre, it can be found as in the next number.

325. To FIND THE CENTRE OF ANY REGULAR POLYGON. Bisect any two sides; and raise perpendiculars from the middle points. The point in which these perpendiculars cut is the centre of the figure.



326. CIRCLE. To FIND THE AREA OR SPACE A CIRCLE COVERS; which is also called its square measure, or squaring the circle. This cannot be found with perfect correctness; but sufficiently correct for all useful purposes.

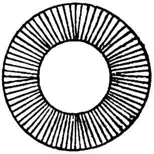
Multiply the radius of the circle by half the length of its circumference.

It also amounts to the same thing if we multiply the diameter of the circle by a quarter of its circumference: or if we multiply the diameter by the circumference and divide the product by 4.

If radius be 7, the half circumference is 22, (No. 255) and 7 times 22 is 154 squares, (inches, or feet, or yards &c.) or, the diameter being 14, the circumference is 44: then $44 \times 14 = 616$: and $\frac{616}{4} = 154$ as before.

327. To FIND THE AREA OF A RING BETWEEN TWO CIRCLES. Find the area of each circle, and subtract the smaller from the larger.

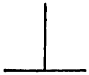
Example. Suppose that the large circle has a diameter of 28 feet; its circumference must be 88 feet long, and its area 616 square feet. The small circle has a diameter of 14 feet: its circumference must be 44 feet long, and its area 154



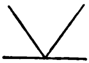
square feet. 154 subtracted from 616 leaves a remainder of 462 square feet which is the area of the ring.

328. Triangle. The Three Angles of any Triangle Are Equal to Two Right Angles.

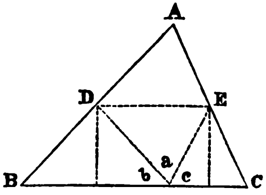
A line meeting another makes two right angles, or angles equal to two right angles. Nos. 47, 53.



Any number of lines falling upon another line, at the same point, make angles equal to two right angles, for they occupy the same space.



Cut out a large paper triangle of any form, and fold down the upper angle A upon the base at *a*, but the fold DE must be made parallel to the base. Then fold over the angle B, to lie beside *a* at *b;* and fold over the angle C, to lie beside *a* at *c*.

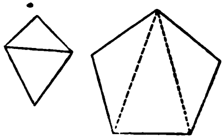


The three angles, *a*, *b* and *c*, lie together at the same point of the straight line BC, and therefore are equal to two right angles.

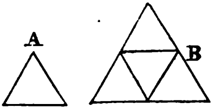
Cut out several more paper triangles, of different shapes, and fold down the angles in the same way. We shall find that the three angles always occupy the space of two right angles.

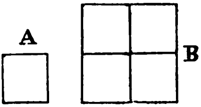
329. To Find how many Right Angles the Angles of Figures are Equal to, which have more than Three Angles. Divide the figure into triangles (No. 274), by

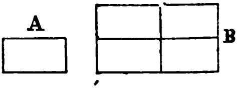
drawing diagonals to one angle. Since the angles of one triangle are equal to two right angles (No. 328), the angles of the figure must be equal to twice as many right angles as the figure contains triangles. The angles of any four-sided figure must therefore be equal to four right angles; the angles of a pentagon to six right angles; the angles of a hexagon to eight right angles, and so on.

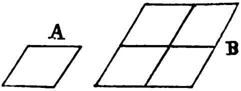


330. To Find the Areas of Figures Compared with the Length of their Sides. If two figures be similar, as A and B, but the sides of one, B, are twice as long as the sides of the other, A, the area of B will be four times the area of A, or A has one quarter the area of B. This is shown by the triangles, squares, oblongs, and rhombs, marked A and B. In this case A is on half the scale of B, or B is on double the scale of A.

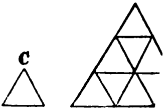




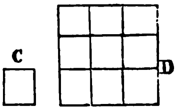




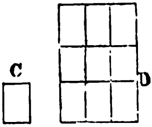
If the sides of one, D, be three times as long as those of the other, C, the area of D will be three times *three*, or nine times as large as the

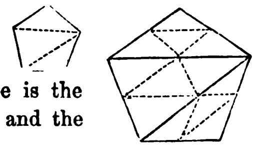


area of C. Here D is on three times the scale of C.



The same thing can be shown to be true of pentagons, or figures with more sides, by dividing each figure into triangles by means of diagonals, and then dividing the triangles of the larger into four, nine, or sixteen triangles, equal respectively to those of the smaller, according as the side of the larger is two, three, or four times as long as the side of the smaller. The upper triangle of the large pentagon contains four triangles equal to the upper triangle of the small pentagon; and the same is the case with the middle triangles, and the lower triangles of both figures.

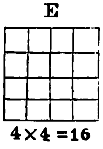


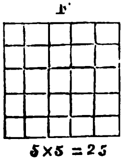


331. By Arithmetic. The comparative size of the areas of two similar figures can be found by multiplying the length of one side of each by itself. Thus, if A be 2 feet and B 4 feet long, the area of A will be 2 × 2 or 4, and of B 4 × 4 or 16 square feet. The area 16 is 4 times as large as the area 4. If the length of C is 2 feet and of D 6 feet, then 2 × 2 is 4, and 6 × 6 is 36; the area 4 is the ninth part of the area 36.

If a side of one similar figure, E, measure 4 feet, and that of another, F, 5 feet,—

4 × 4 = 16 square feet.
5 = 5 = 25 ,, ,,



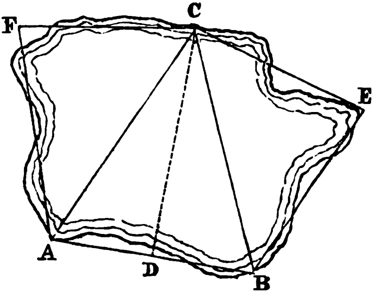


The area of E is to the area of F as 16 compared with 25, or rather more than half as large again.

If a side of one of two similar figures measure 9 feet, and the corresponding side of the other figure 12 feet, the area of the figures is as 81 to 144, or nearly as 3 to 5.

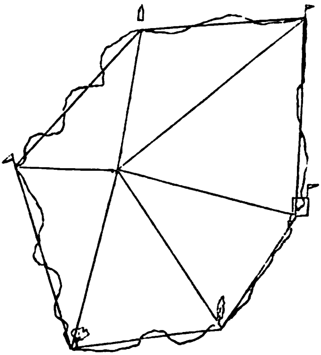
332. MEASUREMENT OF DISTANCES. SURVEYING. Distances across rivers, bogs, ponds, woods, hollows, and long tracts of country, are often measured in this manner.

A line, AB, is first measured on flat ground, and then drawn on paper, an inch to the yard, or to any other scale. The angles A and B, which this line makes with the direction of some object at C, are also measured with an instrument called a theodolite (but they could be measured, though less correctly, with a jointed rule). These angles are then made in the drawing or plan, and the triangle completed. If the perpendicular, CD, measures twenty inches in the plan, the pond must be 20 yards across, if our drawing is on the scale of an inch to the yard, and if our drawing and measurement have been performed correctly.



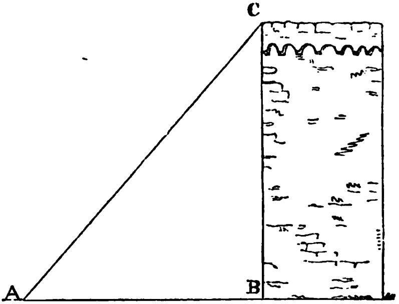
By an extension of this kind of measurement, in which a number of triangles are joined on to each other, districts and countries are measured, and maps of them are made. It is called Surveying. Much skill and knowledge, and many instruments are required to perform it correctly. What is wanted to be measured, is generally divided into triangles, which are drawn of the proper size and form, on paper, and the whole of them form a plan, or map of the country. The above pond is divided into three triangles. We have measured the middle triangle ACB. The triangle to the left is drawn correctly by measuring the angles which AC makes at A and C respectively, with a second object at F. The triangle to the right is drawn after measuring the angles which BC makes at B and C with a third object at E. The whole pond can then be drawn on the plan and measured with almost perfect correctness, for the parts that go beyond the line make up for those which lie within it. By adding more triangles, the ground about the pond could be put into the plan, to as great an extent as we desire. This account of Surveying is very imperfect. It merely gives a general notion of the method; a large book is required for its perfect explanation.

333. The Measurement of Land is a kind of surveying. In this country land is measured by acres and parts of an acre. (See the table of square measure at the end.) An acre may be of any shape. If it is square its side measures about 70 yards; and 640 such acres are required to make up a square mile. 4,900 square yards are contained in an acre. Land is usually measured by means of long wire chains, and 100,000



square links of these chains make up an acre. If land always lay in squares, oblongs or triangles, it could easily be measured; but fields are usually of an irregular shape. The surveyor is therefore obliged to divide the fields into triangles, or other convenient shapes, and measure each of these separately. This field might be divided into six triangles, or in several other ways.

334. MEASUREMENT OF HEIGHTS. Heights, such as those of towers, walls, houses and trees, which cannot be got at for measurement in the usual way, can often be measured by taking a station, as A, on a level with the base, and measuring the distance (in yards) from A to B. A line is then made on paper, to begin a drawing called a plan, one inch (or perhaps less) being drawn for each yard of AB.



The angle at A is then measured by some instrument, and put into the drawing. The angle B is also drawn on the paper at once, as it is known to be a right angle. If the triangle on the plan be now completed, it will be similar to the triangle ABC, which has BC for its vertical side; but its sides will measure one thirty-sixth of the real size if the scale be an inch to the yard. Let the length of BC be measured on the drawing. If it be 37 inches, the tower must be 37 yards high.

335. Measurement of Heights by Shadows, can some*times be* performed with tolerable accuracy. If a pole (or

a man) standing vertically 6 feet high, cast a shadow 3 feet long, and the shadow of a tree or any tall object is found to be 20 feet long, then the tree must be 40 feet high, or twice as tall as its shadow. If the 6-feet pole cast a shadow of 4 feet while the shadow of a house is 20 feet, the house must be 30 feet high, or one-third longer than its shadow.

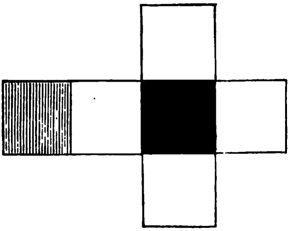
A 10-feet pole has a shadow of 5 feet, and a poplar's shadow is 20 feet; what is the height of the poplar?

A boy 4 feet tall has a shadow of one foot, while a signpost's shadow is five feet; what is the height of the sign-post?

SURFACE MEASURE OF SOLIDS.

336. The surface measurement of solids signifies the measure of their outside, in square measure. Paper that covers every part of a cube exactly, shews the surface measure of the cube.

337. Cube. The surface measure of a cube is six times that of any one face. Therefore, measure one face of the cube, and multiply that by six. The measurement of the outside of a one-inch cube is six square inches. A four-inch cube measures 16 square inches on each face: its entire surface measure is 16 × 6 or 96 square inches. 5 inch, 7-inch, 8-inch, and 10-inch cubes have surfaces of 150, 294, 384 and 600 square feet, which the learner should calculate.

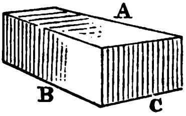


338. To Make a Folding Cube. Draw six squares on paper or card, (No. 243–6) in this manner, and cut out the figure. The black square is the bottom; the shaded square, the top or lid. The squares fold up into a cube, and *may be fixed* by sewing

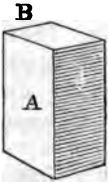
their edges together. The figure can be made in a minute by tracing the six squares round a card square cut out for the purpose. Or a rectangle can be drawn four times as long as it is broad, and divided by lines into four squares; and two squares of the same size must be added to the sides of one of the intermediate squares.

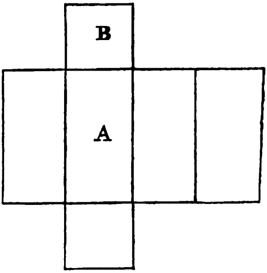
If the figure be made of pasteboard, the outer lines must be scored, or it will not fold.

339. **Parallelopiped and Square Prism.** Find the measurement of one of each pair of opposite faces, viz: A, B and C, and multiply by two.



If the solid be a prism, which has four equal faces, and two equal bases, measure one face A, and multiply by four, then measure one base B, multiply that by two; and add this product to the former.





A brick is a parallelopiped like the first of the above diagrams. The largest face A, measures 10 inches by 5 inches; *the face B* measures 10 inches by 3 inches; and the face C, *5 inches* by 3 inches.

Faces. Sq. In.

A, 10 × 5 = 50; and 50 × 2 = 100 surface measure of the two largest faces.

B, 10 × 3 = 30; and 30 × 2 = 60 surface of two middle sized faces.

C, 5 × 3 = 15; and 15 × 2 = 30 surface of two smallest faces.

190 total surface measure of a brick in square ins.

What is the surface measure of a box 2 feet by 3, and 4 feet deep?

" " log 3 feet by 4, and 9 feet long?

" " cistern 5 feet by 7, and 12 feet deep?

" " room 12 feet by 16, and 10 feet high?

Books, boxes, houses, rooms, logs of wood, blocks of stone, and innumerable other articles are of this shape; which therefore requires to be measured more frequently than any other.

Example of a square prism. A square pillar 8 feet long, having 2 feet square bases. See second pair of diagrams.

A, one face, measures 8 feet × 2 feet = 16; and 16 × 4 = 64 surface measure of 4 faces or sides.

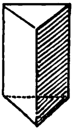
B, one base or end, 2 feet × 2 feet = 4; and 4 × 2 = 8 ditto of two ends.

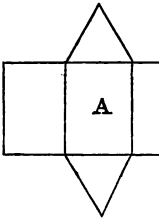
72 square feet the entire surface measure.

92

340. Folding Parallelopipeds and Square Pr can be easily made by drawing and cutting out large fig like the right figure in the above diagrams of the p which will fold up like the diagram of the cube in No.

341. Prism. A prism has three faces or sides, o greater number. It can have only two ends or bases. Since the faces of a regular prism are equal, find the measurement of one face and multiply that by the number of the faces: to the product add double the measurement of one ba





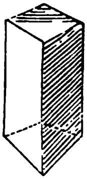
342. Triangular prism. The two diagrams shew th face of a triangular or three-sided prism, opened ou then folded up round the solid.

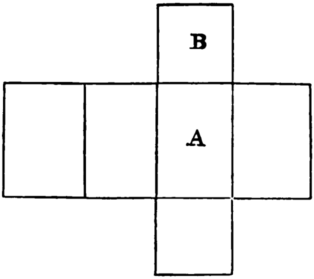
A measures 8 feet by 6 feet = 48 feet square mea 48 × 3 = 144, the square measure of the three faces. triangular base has an altitude of about 5 feet, and it measures 6 feet: then 6 × 5 = 30, and 30 divided b the square measure of the triangle. The two trian bases measure 15 × 2 = 30. The entire surface me of this prism is therefore 144 and 30, or 174 square This solid can be made for *folding*, by drawing a larg gram like the second, and cutting it out, folding it up fixing the outer edges together.

343. The Four-sided Right Prism has four rectan faces of equal measurement, and two equal square e bases.

Let A be 6 feet by 4; then, 6 × 4 = 24; and 2 = 96, the four faces.

Let B be 4 feet square; then, 4 × 4 = 16; and 16 × 2 = 32, the bases. 96 and 32 are 128 square feet, the surface measure of the solid.





344. The folding of this solid merely requires an oblong of paper to be folded into four, and two bases of proper size to be added.

345. Prisms with five sides, or six sides, or any greater number, can have their surface measure calculated in the same manner. The first requires five equal faces, and two equal pentagons for bases. The second has six equal faces, and two equal hexagons for bases.

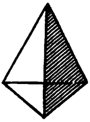
346. These solids can be folded up at once; the one, by folding an oblong bit of paper into five equal narrow oblongs; the other, by folding the paper into six such oblongs; and adding bases of the proper form and size.

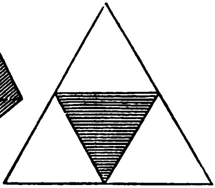
347. When prisms are irregular, that is, when their sides or bases are not equal, each side or base must be calculated separately, and the whole must be added together.

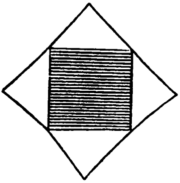
348. Pyramids. As a pyramid has three triangles, or any greater number, for faces, and a right-lined figure for a base, the surface measure is found by measuring one triangle and multiplying its area by the number of the triangular

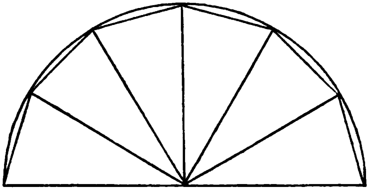
faces; and then adding to the product, the measuremen the base.

349. This rule, and the way to fold up these solids, shewn by the following diagrams.









350. **The Three-sided Pyramid** is made by folding equilateral triangle into four. The dark triangle is the ba

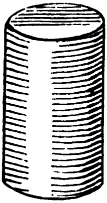
351. **The Four-sided Pyramid** has four similar triang drawn on the sides of a square base.

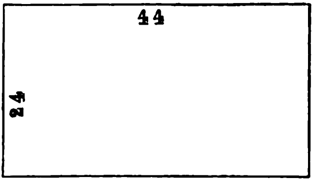
352. A pyramid of any number of faces can be fold by dividing a large semicircle into the proper number sectors for faces, cutting off their curved bases, and inserti the proper figure as the base of the solid. The last diagr folds up into a six-sided pyramid. Three of its triang would fold into a triangular pyramid: four of them, int square pyramid: five, into a pentagonal pyramid: a eight such, into an octagonal pyramid.

353. A figure for folding up into a pyramid can also *traced at* a central point, with the same isosceles triang

repeated as many times as the pyramid is to have sides. But the triangles must not be made completely round the point, as, in that case, the apex cannot be raised without making the extreme triangles overlap.

354. Cylinder. Multiply the circumference of a base by the altitude of the curved surface. To this product add twice the square measure of one of the bases.

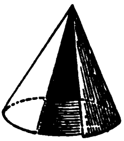


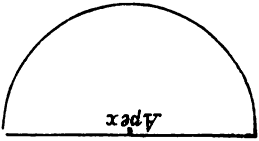


A cylinder measures 44 feet in the circumference of its base; its altitude is 24 feet. The surface of the curved part is therefore 44 × 24 or 1,056 square feet. Each circular base measures the half of 44 multiplied by the radius 7. 22 × 7 = 154 one base. 154 × 2 = 308 both bases. 308 + 1,056 = 1,364 square feet, the total surface measure.

355. Folding up a Cylinder. Any paper oblong will curve into a cylinder, which will be made perfect by fixing in a paper circle, of the right size, at each end or base. The paper oblong for the above cylinder must measure 44 by 24, as is shewn in the diagram.

356. Cone. The surface measure of the curved part is found by multiplying the circumference of the base by half the slanting height; and the whole surface measure is obtained by adding to this product the mea-





sure of the flat base. If the circumference be 22 and the slanting height 10, the curved surface will measure 110, to which add the measure of the base, 38½.

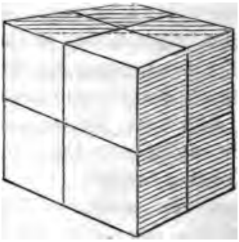
357. A large paper or card semicircle will bend into a cone. The centre of the circle is the top or apex of the cone; and the half circumference is the circular base. A card circle of the proper size must be inserted at the bottom to complete the cone. Any sector of a circle will also curve into a cone.

358. Sphere. Multiply the circumference of a large circle by its diameter. If a sphere's circumference be 22 inches, its diameter is 7 inches; and the surface measure $22 \times 7 = 154$ square inches.

SOLID MEASURE.

359. Cubic Measure, or Measure of Capacity. Solid measure is also called cubic measure, because we have to find how many cubes of a certain size (as cubic inches or cubic feet) are contained in any solid, or in any hollow vessel that we want to measure.

360. Cube. To Measure a Two-inch Cube. An inch cube is one inch long, one inch broad, and one deep. A two-inch cube is two inches every way. It can be made up with two layers of one-inch cubes, each layer consisting of four cubes: it is therefore equal to eight one-inch cubes.



A 3-inch cube has 3 layers of 9 each or 27 cubic inches.
A 4 ,, 4 ,, 16 ,, 64 ,,
A 5 ,, 5 ,, 25 ,, 125 ,,

The rule is, multiply the length by the breadth and the product by the thickness. Then the

2-inch cube is $2 \times 2 = 4$; and $4 \times 2 = 8$ cubic inches.
3 ,, $3 \times 3 = 9$; ,, $9 \times 3 = 27$,,
4 ,, $4 \times 4 = 16$; ,, $16 \times 4 = 64$,,
5 ,, $5 \times 5 = 25$; ,, $25 \times 5 = 125$,,

A cubic yard is 27 cubic feet, as $3 \times 3 = 9$, and $9 \times 3 = 27$. A cubic foot is 1,728 cubic inches, as $12 \times 12 = 144$, and $144 \times 12 = 1{,}728$.

361. Parallelopipeds and Four-sided Prisms. The rule is the same for calculating the solid contents of these shapes. If a block of stone be 4 feet long, 3 feet broad, and 5 feet deep, its solid contents are 60 cubic feet; $4 \times 3 = 12$; and $12 \times 5 = 60$.

A room 8 feet high, 12 feet long and 10 feet broad, has 960 cubic feet. $8 \times 12 = 96$; and $96 \times 10 = 960$.

A brick measures 10 inches by 5 inches, and 3 inches. Its solid measure is $10 \times 5 = 50$; and $50 \times 3 = 150$ cubic inches.

362. Prism. To Find the Cubic Contents of a Prism. Find the area, or square measure of the base; and multiply that by the altitude of the solid. Example. A triangular prism, with right-angled triangles for bases, having short sides 4 feet;—altitude of prism 12 feet. Surface of base 8 feet: solid contents 8×12 or 96 cubic feet.

363. Pyramid. To Find the Cubic Contents of any Pyramid. Find the square measure or area of the base, and multiply that by the altitude; then divide the product by 3: because a pyramid has one-third the bulk of a prism on the same base, and of the same altitude or vertical

height. Example. A four-sided pyramid of 6 feet and 9 feet altitude. Base 36 square feet. Solid co $36 \times 9 = 324$; and $\frac{324}{3} = 108$ cubic feet.

364. CYLINDER. TO FIND ITS CUBIC CONTENTS. the area or square measure of the circular base, and n that by the altitude. Example. Circumference of a pillar 44 inches; altitude 4 feet or 48 inches: area 154 square inches: (No. 326.) $154 \times 48 = 7{,}39$ inches, or 4 cubic feet and 480 cubic inches.

365. CONE. TO FIND ITS CUBIC CONTENTS. F area or square measure of the base; multiply that altitude (or vertical height) and divide the produ A cone is one-third the bulk of a cylinder on the sa and of the same altitude. Example. Circular ference 44 inches; altitude 5 feet or 60 inches.

base 154 square inches. $154 \times 60 = 9{,}240$; an

$= 1080$ cubic inches.

366. SPHERE. TO FIND ITS CUBIC CONTENTS. M its surface measure (No. 358) by the radius, and di product by 3. Example. If its large circle be 22 the surface measures 154 square inches. Its radi inches, $154 \times 3\frac{1}{2} = 539$; and $\frac{539}{3} = 179\frac{2}{3}$ cubic i

TABLES.

MEASURES OF LENGTH.

12	inches	make	1 foot.
3	feet	,,	1 yard.
6	yards	,,	1 fathom.
5½	,,	,,	1 pole, perch, or rod.
220	,, or,	,,	1 furlong.
40	poles		
1,760	yards, or	,,	1 mile.
80	chains, or		
8	furlongs		
3	miles	,,	1 league.
69$\frac{1}{15}$	miles, or	,,	1 degree.
60	geogrphc. miles		
7.92	inches	,,	1 link.
66	feet, or	,,	1 chain.
100	links		
2¼	inches	,,	1 nail.
4	nails	,,	1 quarter yard.
4	quarters	,,	1 yard.
5	quarters	,,	1 English ell.
4	inches	,,	1 hand.
9	,,	,,	1 span.
2½	feet	,,	1 pace (military).
39.37	inches	,,	1 mètre (French).

SURFACE OR SQUARE MEASURE.

144	sq. inches . . .	make	1 sq. foot.
9	sq. feet	,,	1 sq. yard.
1,296	sq. inches . . .	,,	1 ,,
30¼	sq. yards . . .	,,	1 sq. pole or pe
16	sq. poles.	,,	1 sq. chain.
40 1,210	sq. poles, or sq. yards . .	,,	1 sq. rood.
4 10 160 100,000	roods, or sq. chains, or sq. poles, or sq. links .	,,	1 acre.
640	sq. acres . . .	,,	1 sq. mile.

The side of a square acre is nearly 69$\frac{4}{7}$ yards.

2 acres, 1 rood, and 30 poles, is nearly 1 hecataire (Fre

SOLID OR CUBIC MEASURE.

1,728	cubic inches . . .	make	1 cubic foo
27	,, feet	,,	1 ,, yar
46,656	,, inches.	,,	1 ,, ,,
277¼	,, ,, ...	,,	1 gallon.
42	,, feet	,,	1 ton, ship
5	,, ,, ...	,,	1 barrel, bu
40 50	,, ,, rough timber ,, ,, hewn ,,	,,	1 load.
35.316	,, ,, ...	,,	1 stere (Fr

MEASURE OF LIQUIDS AND DRY GOODS NOT HEAPED.

4	gills	make		1 pint.
2	pints	,,		1 quart.
4	quarts	,,		1 gallon.
2	gallons	,,	Dry.	1 peck.
4	pecks	,,		1 bushel.
8	bushels	,,		1 quarter.
5	quarters	,,		1 load.
3	bushels, heaped	,,		1 sack.
12	sacks, heaped	,,		1 chaldron.
63	gallons	,,	Wine & Spts.	1 hogshead.
84	,,	,,		1 puncheon.
2	hogsheads	,,		1 pipe.
2	pipes	,,		1 tun.
36	gallons	,,	Ale & Beer.	1 barrel.
1½	barrels	,,		1 hogshead.
3 2	barrels, or hogsheads	,,		1 butt.
2	butts	,,		1 tun.
2½	pints, nearly	,,		1 litre (French).

WEIGHTS.—AVOIRDUPOISE.

16	drachms	make	1 ounce.
16	ounces	,,	1 pound, lb.
28	pounds	,,	1 quarter, qr.
4 112	quarters, or pounds	,,	1 hundred weight, cwt.
20	hundred weight	,,	1 ton.
7,000	grains	,,	1 pound.

WEIGHTS—*continued.*

TROY WEIGHT.

24 grains . . . . make 1 pennyweight, dwt.
20 pennyweights . . „ 1 ounce, oz.
12 ounces . . . . „ 1 pound, ℔.
15.444 grains . . . . „ 1 gramme (French).

APOTHECARIES WEIGHT.

20 grains, gr. . . . make 1 scruple, ℈.
3 scruples. . . . „ 1 drachm, ʒ.
8 drachms . . . „ 1 ounce, ℥.
12 ounces . . . . „ 1 pound, ℔.

ANGULAR MEASURE, OR DIVISIONS OF THE CIRCLE.

60 seconds. . . . make 1 minute, 1′.
60 minutes . . . „ 1 degree, 1°.
30 degrees. . . . „ 1 sign.
90 „ . . . . „ 1 quadrant.
360 „ . . . . „ 1 circumference.

MISCELLANIES.

BOOKS. 2 leaves = 1 sheet folio; 4 leaves = 1 sheet quarto, 4to.; 8 leaves = 1 sheet octavo, 8vo.; 12 leaves = 1 sheet duodecimo, 12mo.; 18 leaves = 1 sheet 18mo.

BREAD. Quartern loaf = 4lbs. 5½oz.

MISCELLANEOUS—continued.

Bricks. [illegible] = [illegible]

Butter. 56lbs. = 1 firkin; 84lbs. = 1 tub.

Calico. 28 yards = 1 piece.

Coal. 1 bushel = [illegible]

Digging. 20 cubic feet of sand, or [illegible] or 17 cubic feet of clay = 1 ton; [illegible] 18 heaped bushels before digging, and 27 when dug = 1 load.

Fish. 1 stone = [illegible]lbs.

Flooring, Roofing, etc. 100 square feet = 1 square.

Flour. 14lbs. = 1 peck or stone; 56lbs. = 1 bushel; 280lbs. or 5 bushels = 1 sack; 196lbs. = 1 barrel.

Glass. 1 stone = 5lbs.

Grain. Wheat. About 60lbs. = 1 bushel.
Barley. .. 47lbs. = 1 ,,
Oats. .. 38lbs. = 1 ,,

Gross. 12 dozen; great gross = 12 gross.

Hay. 56lbs. old or 60lbs. new = 1 truss; 36 trusses = 1 load.

Hops. 1 pocket = 112lbs.

Hemp. 32lbs. = 1 stone.

Lead. 19½ cwt. = 1 fodder.

Meat. 8lbs. = 1 stone.

Mile-League:—

Irish mile	2,240	yards
Scotch ,, ...	1,984	,,
German ,, short . . .	6,859	,,
,, ,, long . . .	10,126	,,
Prussian ,, ...	8,237	,,
French league-post . .	4,263	,,
,, ,, geographical	4,860	,,

MISCELLANIES—*continued.*

Muslin. 1 piece = 10 yards.

Palm. 3 inches.

Paper. 24 sheets = 1 quire; 20 quires = 1 ream.

Potatoes. 40 bushels = 1 ton.

Raisins. 1 barrel = 112lbs.

Rice. 1 bag = 168lbs.

Salt. 1 bushel = 56lbs.

Straw. 36lbs. = 1 truss; 36 trusses = 1 load.

Stone. Horseman's weight = 14lbs.

Tea. 1 chest = 14lbs.

Tiles. 1,000 = 1 load.

Water. 1 gallon weighs 10lbs. avoirdupoise; 1 cubic inch = 252.458 grains troy.

Wood. 128 cubic feet = 1 cord.

Wool. 14lbs. = 1 stone; 28lbs. = 1 tod; 240lbs. = 1 pack.

FINIS.

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